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<tr>
<td>1</td>
<td>Multiple Choice</td>
<td>Create equations that describe numbers or relationships.</td>
<td>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (A.CED.4)</td>
<td>D</td>
<td>1 point</td>
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<tr>
<td>2</td>
<td>Multi-Select Item</td>
<td>Extend the properties of exponents to rational exponents.</td>
<td>Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N.RN.2)</td>
<td>A, C, E</td>
<td>1 point</td>
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<tr>
<td>3</td>
<td>Multiple Choice</td>
<td>Write expressions in equivalent forms to solve problems.</td>
<td>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (A.SSE.3) a. Factor a quadratic expression to reveal the zeros of the function it defines.</td>
<td>C</td>
<td>1 point</td>
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<td>4</td>
<td>Multiple Choice</td>
<td>Perform arithmetic operations on polynomials.</td>
<td>Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1)</td>
<td>D</td>
<td>1 point</td>
</tr>
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<td>5</td>
<td>Equation Item</td>
<td>Use coordinates to prove simple geometric theorems</td>
<td>Find the point on a directed line segment between two given points that partitions the segment in a</td>
<td>---</td>
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<tr>
<td></td>
<td></td>
<td>algebraically.</td>
<td>given ratio. (G.GPE.6)</td>
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<td>6</td>
<td>Multiple Choice</td>
<td>Prove theorems involving similarity.</td>
<td>Use congruence and similarity criteria for triangles to solve problems and to prove relationships in</td>
<td>A</td>
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<tr>
<td></td>
<td></td>
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<td>geometric figures. (G.SRT.5)</td>
<td></td>
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<tr>
<td>7</td>
<td>Short Response</td>
<td>Understand independence and conditional probability,</td>
<td>Recognize and explain the concepts of conditional probability and independence in everyday language and</td>
<td>---</td>
<td>2</td>
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<tr>
<td></td>
<td></td>
<td>and use them to interpret data.</td>
<td>everyday situations. For example, compare the chance of having lung cancer if you are a smoker with</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>the chance of being a smoker if you have lung cancer. (S.CP.5)</td>
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<tr>
<td>8</td>
<td>Multiple Choice</td>
<td>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</td>
<td>Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. (S.CP.6)</td>
<td>A</td>
<td>1 point</td>
</tr>
<tr>
<td>9</td>
<td>Multiple Choice</td>
<td>Understand independence and conditional probability, and use them to interpret data.</td>
<td>Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). (S.CP.1)</td>
<td>A</td>
<td>1 point</td>
</tr>
<tr>
<td>10</td>
<td>Graphic Response</td>
<td>Prove geometric theorems.</td>
<td>Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G.CO.11)</td>
<td>---</td>
<td>1 point</td>
</tr>
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<tr>
<td>11</td>
<td>Multiple Choice</td>
<td>Define trigonometric ratios, and solve problems involving right triangles.</td>
<td>Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)</td>
<td>D</td>
<td>1 point</td>
</tr>
<tr>
<td>12</td>
<td>Multiple Choice</td>
<td>Prove theorems involving similarity.</td>
<td>Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)</td>
<td>B</td>
<td>1 point</td>
</tr>
<tr>
<td>13</td>
<td>Equation Item</td>
<td>Define trigonometric ratios, and solve problems involving right triangles.</td>
<td>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>14</td>
<td>Multiple Choice</td>
<td>Understand and apply theorems about circles.</td>
<td>Prove that all circles are similar. (G.C.1)</td>
<td>B</td>
<td>1 point</td>
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<tr>
<td>15</td>
<td>Equation Item</td>
<td>Analyze functions using different representations.</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).  For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.  (F.IF.9)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>16</td>
<td>Multiple Choice</td>
<td>Interpret functions that arise in applications in terms of the context.</td>
<td>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.  Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.  (F.IF.4)</td>
<td>C</td>
<td>1 point</td>
</tr>
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</table>
| 17           | Multiple Choice | Analyze functions using different representations. | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.  
\((F.IF.8)\)  
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | D          | 1 point |
| 18           | Equation Item   | Prove and apply trigonometric identities. | Prove the Pythagorean identity \(\sin^2(\theta) + \cos^2(\theta) = 1\), and use it to find \(\sin(\theta)\), \(\cos(\theta)\), or \(\tan(\theta)\) given \(\sin(\theta)\), \(\cos(\theta)\), or \(\tan(\theta)\) and the quadrant of the angle.  
\((F.TF.8)\) | ---         | 1 point |
| 19           | Multiple Choice | Build new functions from existing functions. | Identify the effect on the graph of replacing \(f(x)\) by \(f(x) + k\), \(kf(x)\), \(f(kx)\), and \(f(x + k)\) for specific values of \(k\) (both positive and negative); find the value of \(k\) given the graphs.  
Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.  
\((F.BF.3)\) | B          | 1 point |
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<tr>
<td>20</td>
<td>Equation Item</td>
<td>Interpret functions that arise in applications in terms of the context.</td>
<td>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. ((F.IF.6))</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>21</td>
<td>Table Item</td>
<td>Solve systems of equations.</td>
<td>Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line (y = -3x) and the circle (x^2 + y^2 = 3). ((A.REI.7))</td>
<td>---</td>
<td>2 points</td>
</tr>
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</table>
The equation shown is used to find the force of gravity, \( F \), between two objects, where

- \( G \) is the gravitational constant,
- \( m_1 \) and \( m_2 \) are the masses of the two objects, and
- \( r \) is the distance between the two objects.

\[
F = \frac{Gm_1m_2}{r^2}
\]

Which equation correctly shows the distance between the two objects?

A) \( r = \frac{\sqrt{F}}{Gm_1m_2} \)

B) \( r = \frac{\sqrt{Gm_1m_2}}{F} \)

C) \( r = \sqrt{\frac{F}{Gm_1m_2}} \)

D) \( r = \sqrt{\frac{Gm_1m_2}{F}} \)

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (A.CED.4)
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have taken the square root of $F$ and $r^2$ but not the other terms, and he or she may have also confused multiplying with dividing when rearranging the equation to find $r$.

Rationale for Option B: This is incorrect. The student may only have taken the square root of the terms on the right side of the equation before rearranging to find $r$.

Rationale for Option C: This is incorrect. The student may have taken the square root of all the terms but confused multiplication with division when rearranging to find $r$.

Rationale for Option D: Key – The student correctly rearranged the formula, $F = \frac{G(m_1)(m_2)}{r^2}$, to find $r$. The first step would be to multiply both sides by $r^2$ which would result in $(r^2)(F) = G(m_1)(m_2)$. Then the student would divide each side by $F$ resulting in $r^2 = \frac{G(m_1)(m_2)}{F}$. Finally, he or she would take the square root of both sides to get $r = \sqrt{\frac{G(m_1)(m_2)}{F}}$. 

The equation shown is used to find the force of gravity, $F$, between two objects, where

- $G$ is the gravitational constant,
- $m_1$ and $m_2$ are the masses of the two objects, and
- $r$ is the distance between the two objects.

$$F = \frac{Gm_1m_2}{r^2}$$

Which equation correctly shows the distance between the two objects?

(A) $r = \frac{\sqrt{F}}{Gm_1m_2}$

(B) $r = \frac{\sqrt{Gm_1m_2}}{F}$

(C) $r = \sqrt{\frac{F}{Gm_1m_2}}$

(D) $r = \sqrt{\frac{Gm_1m_2}{F}}$
Question 2

Select all of the expressions that are equivalent to $16^{\frac{5}{2}}$.

- $4^5$
- $8^5$
- $\sqrt{16^5}$
- $\sqrt[5]{16^2}$
- $(16^2)(16^{\frac{1}{3}})$
- $(16^{\frac{5}{2}})(16^{\frac{1}{3}})$

Points Possible: 1

Content Cluster: Extend the properties of exponents to rational exponents.

Content Standard: Rewrite expressions involving radicals and rational exponents using the properties of exponents. \((N.RN.2)\)
Scoring Guidelines

Rationale for First Option: **Key** – The student correctly selected an equivalent expression. According to the property of exponents
\[
16^{\frac{5}{2}} = 16^2 \cdot 16^{\frac{1}{2}}. \quad \text{Since} \quad 16^2 = (4^2)^2 = 4^4 \quad \text{and} \quad 16^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} = 4, \quad \text{then}
\]
\[
16^2 \cdot 16^{\frac{1}{2}} = 4^4 \cdot 4 = 4^5 \quad \text{and} \quad 16^{\frac{5}{2}} = 4^5.
\]

Rationale for Second Option: This is incorrect. The student may have divided the 16 by the 2 in the denominator of the exponent instead of taking the square root of 16.

Rationale for Third Option: **Key** – The student correctly selected an equivalent expression because expressions in the form \(x^\frac{b}{a}\) can be written as \(\sqrt[\frac{a}{b}]x\) so that
\[
16^{\frac{5}{2}} = \sqrt[\frac{2}{5}]16^5.
\]

Rationale for Fourth Option: This is incorrect. The student may have recognized that numbers with fraction exponents can be written with radicals, but reversed the meaning of the numerator and denominator and used the numerator as the radical’s index instead of the denominator.

Rationale for Fifth Option: **Key** – The student correctly selected an equivalent expression because the product of two exponential expressions with the same bases is the exponential expression with the unchanged base and the sum of exponents. In this case, the sum of the exponents is 2.5 or \(\frac{5}{2}\) in the given expression, so
\[
16^{\frac{5}{2}} = 16^{\frac{2}{2} + \frac{2}{2} + \frac{1}{2}} = 16^2 \cdot 16^{\frac{1}{2}}.
\]

Rationale for Sixth Option: This is incorrect. The student may have incorrectly converted the improper fraction \(\frac{5}{2}\) to 5 and \(\frac{1}{2}\) instead of 2 and \(\frac{1}{2}\), and applied the product property of exponents \(16^{5.5} = 16^{5+\frac{1}{2}} = 16^5 \cdot 16^{\frac{1}{2}}\).
Sample Response: 1 point

Select all of the expressions that are equivalent to $16^{\frac{5}{2}}$. 

- $4^5$
- $8^5$
- $\sqrt[5]{16^5}$
- $\sqrt[5]{16^2}$
- $(16^2)(16^{\frac{1}{3}})$
- $(16^5)(16^{\frac{1}{3}})$
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Question 3

Question and Scoring Guidelines
Question 3

Which correctly factored form of the function $f(x) = 36x^2 + 15x - 6$ can be used to identify the zeros?

A. $f(x) = (4x - 1)(3x + 2)$
B. $f(x) = (12x - 2)(3x + 3)$
C. $f(x) = 3(4x - 1)(3x + 2)$
D. $f(x) = 3(12x - 2)(3x + 3)$

Points Possible: 1

Content Cluster: Write expressions in equivalent forms to solve problems.

Content Standard: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (A.SSE.3)

a. Factor a quadratic expression to reveal the zeros of the function it defines.
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have factored the greatest common factor from the expression and then left it out from the final answer since it is not a binomial. The student may have also failed to use multiplication to verify that it is not equivalent to the quadratic expression in the given $f(x)$.

Rationale for Option B: This is incorrect. The student may have factored only the squared term, $36x^2$, and constant term, $-6$, when selecting option B. The student may have also failed to use multiplication to verify that it is not equivalent to the quadratic expression in the given $f(x)$.

Rationale for Option C: Key – The student chose the completely factored form of the given expression by stating that $36x^2 + 15x - 6 = 3(12x^2 + 5x - 2) = 3(4x - 1)(3x + 2)$. Next, the student may have first used the distributive property and then multiplication to verify if the expression in option C is equivalent to the quadratic expression in the given $f(x)$.

Rationale for Option D: This is incorrect. The student may have recognized that the expression has a greatest common factor of 3, but then may have factored just the squared term, $36x^2$, and constant term, $-6$, of the given quadratic expression $f(x)$. The student may have also failed to use first the distributive property and then multiplication to verify that it is not equivalent to the quadratic expression in the given $f(x)$.

Sample Response: 1 point

Which correctly factored form of the function $f(x) = 36x^2 + 15x - 6$ can be used to identify the zeros?

- $f(x) = (4x - 1)(3x + 2)$
- $f(x) = (12x - 2)(3x + 3)$
- $f(x) = 3(4x - 1)(3x + 2)$
- $f(x) = 3(12x - 2)(3x + 3)$
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Question 4

Question and Scoring Guidelines
Question 4

An expression is shown.

\[(2x - 3) + [4x(3x + 2)]\]

Which expression is equivalent to the given expression?

A) \[9x - 1\]
B) \[14x + 5\]
C) \[12x^2 + 2x - 1\]
D) \[12x^2 + 10x - 3\]

Points Possible: 1

Content Cluster: Perform arithmetic operations on polynomials.

Content Standard: Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1)
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have added all the linear terms, 2x, 4x and 3x together, without applying the distributive property, and then added all the constant terms, −3 and 2, together.

Rationale for Option B: This is incorrect. The student may have distributed the 4 instead of 4x which resulted incorrectly in 2x − 3 + 12x + 8 and then combined like terms.

Rationale for Option C: This is incorrect. The student may have distributed the 4x term only to the 3x term, not the 2, thereby getting (2x − 3) + (12x^2 + 2). Then after combining like terms, the result would be 12x^2 + 2x − 1.

Rationale for Option D: Key – The student correctly performed operations with polynomials as (2x − 3) + [4x(3x + 2)] = (2x − 3) + [12x^2 + 8x] = 12x^2 + 10x − 3.

Sample Response: 1 point
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Question 5

Question and Scoring Guidelines
Question 5

Line segment AB has endpoints A (−1.5, 0) and B (4.5, 8). Point C is on line segment AB and is located at (0, 2).

What is the ratio of \( \frac{AC}{CB} \)?

Points Possible: 1

Content Cluster: Use coordinates to prove simple geometric theorems.

Content Standard: Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)

Scoring Guidelines

Exemplar Response

- \( \frac{1}{3} \)

Other Correct Responses

- Any equivalent ratio
- Any decimal value 0.33, 0.333, 0.3333, etc.

For this item, a full-credit response includes:

- The correct ratio (1 point).
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Question 5

Sample Responses
Sample Response: 1 point

Line segment AB has endpoints A (-1.5, 0) and B (4.5, 8). Point C is on line segment AB and is located at (0, 2). What is the ratio of $\frac{AC}{CB}$?

$\frac{1}{3}$
Notes on Scoring

This response earns full credit (1 point) because it shows a correct ratio of lengths of $\overrightarrow{AC}$ and $\overrightarrow{CB}$, or $\frac{2.5}{7.5}$ or $\frac{1}{3}$.

There are different ways to find the ratio. One of them is to find the lengths of $\overrightarrow{AC}$ and $\overrightarrow{CB}$ using the distance formula,

\[
\overrightarrow{AC} = \sqrt{((-1.5 - 0)^2 + (0 - 2)^2)}
= \sqrt{(-1.5)^2 + (-2)^2}
= \sqrt{6.25}
= 2.5
\]

and

\[
\overrightarrow{CB} = \sqrt{((0 - 4.5)^2 + (2 - 8)^2)}
= \sqrt{(-4.5)^2 + (-6)^2}
= \sqrt{56.25}
= 7.5.
\]

The ratio of $\frac{\overrightarrow{AC}}{\overrightarrow{CB}}$ is $\frac{2.5}{7.5}$ or $\frac{1}{3}$.

Another way is based on the notion that the ratio of $\frac{\overrightarrow{AC}}{\overrightarrow{CB}}$ is equal to the ratio of the corresponding lengths of the line segments along the x-axis: $\frac{0 - (-1.5)}{4.5 - 0}$ or $\frac{1.5}{4.5}$ or $\frac{1}{3}$. The same would be true about the corresponding lengths of the line segments along the y-axis:

\[
\frac{(2 - 0)}{(8 - 2)}
= \frac{2}{6}
= \frac{1}{3}.
\]

The ratio of $\frac{1}{3}$ expressed in form of a decimal value as 0.33, 0.333 and etc. are accepted.
Sample Response: 1 point

Line segment AB has endpoints A (-1.5, 0) and B (4.5, 8). Point C is on line segment AB and is located at (0, 2).

What is the ratio of \( \frac{AC}{CB} \)?

\[
\frac{2.5}{7.5}
\]
Notes on Scoring

This response earns full credit (1 point) because it shows a correct ratio of lengths of $\overline{AC}$ and $\overline{CB}$, $\frac{AC}{CB}$, or $\frac{1}{3}$.

There are different ways to find the ratio. One of them is to find the lengths of $\overline{AC}$ and $\overline{CB}$ using the distance formula,

$$AC = \sqrt{((-1.5 - 0)^2 + (0 - 2)^2)}$$
$$= \sqrt{(-1.5)^2 + (-2)^2}$$
$$= \sqrt{6.25}$$
$$= 2.5$$

and

$$CB = \sqrt{((0 - 4.5)^2 + (2 - 8)^2)}$$
$$= \sqrt{(-4.5)^2 + (-6)^2}$$
$$= \sqrt{56.25}$$
$$= 7.5.$$ 

The ratio of $\frac{AC}{CB}$ is $\frac{2.5}{7.5}$. 
Sample Response: 0 points

Line segment AB has endpoints A \((-1.5, 0)\) and B \((4.5, 8)\). Point C is on line segment AB and is located at \((0, 2)\).

What is the ratio of \(\frac{AC}{CB}\)?

10

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect ratio of \(\frac{AC}{CB}\) as 10.

The student may have found the length of the line segment AB using the distance formula instead of finding the ratio of AC to CB,

\[
AB = \sqrt{((4.5 - 1.5)^2 + (8 - 0)^2)}
\]

\[
= \sqrt{(6)^2 + (8)^2}
\]

\[
= \sqrt{100}
\]

\[= 10.\]
Sample Response: 0 points

Line segment AB has endpoints A (−1.5, 0) and B (4.5, 8). Point C is on line segment AB and is located at (0, 2).

What is the ratio of $\frac{AC}{CB}$?

0.148134

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect ratio of $\frac{AC}{CB}$ as 0.148134.
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Question 6

Question and Scoring Guidelines
Which triangle must be similar to triangle XYZ?

A. a triangle with two angles that measure 40°
B. a triangle with angles that measure 40° and 60°
C. a scalene triangle with only one angle that measures 100°
D. an isosceles triangle with only one angle that measures 40°

Points Possible: 1

Content Cluster: Prove theorems involving similarity.

Content Standard: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G.SRT.5)
Scoring Guidelines

Rationale for Option A: **Key** – The student correctly realized that the third angle of the given triangle is 40 degrees and applied the AA criterion to identify a similar triangle since the sum of the angles in any triangle is 180°.

Rationale for Option B: This is incorrect. The student may have incorrectly found the measure of angle Y by subtracting 40° from 100° to get 60° (100 – (40)= 60), instead of subtracting the sum of 40° and 100° from 180° (180 – (100+40)=40), and then applied the AA criterion. The correct angle measure of Y is 40° instead of 60°.

Rationale for Option C: This is incorrect. The student may have made an assumption that triangle XYZ looks scalene and then incorrectly concluded that any two scalene triangles that have one congruent angle that measures 100° are similar.

Rationale for Option D: This is incorrect. The student may have incorrectly concluded that all isosceles triangles with one angle of 40° should have another base angle of 40°. However an isosceles triangle could have a vertex angle of 40°, which leaves its base angle measurements as 70° and 70°. This triangle would not be similar to triangle XYZ. A similar triangle would have to have both base angles be 40° not just one of them.
Triangle XYZ is shown.

Which triangle must be similar to triangle XYZ?

- a triangle with two angles that measure 40°
- a triangle with angles that measure 40° and 60°
- a scalene triangle with only one angle that measures 100°
- an isosceles triangle with only one angle that measures 40°
Integrated Math II
Spring 2017 Item Release

Question 7

Question and Scoring Guidelines
Question 7

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

<table>
<thead>
<tr>
<th></th>
<th>Arrives at School on Time</th>
<th>Arrives at School Late</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goes to Bed by 10:00 p.m.</td>
<td>72</td>
<td>8</td>
</tr>
<tr>
<td>Goes to Bed After 10:00 p.m.</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Points Possible: 2

Content Cluster: Understand independence and conditional probability, and use them to interpret data.

Content Standard: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)
## Scoring Guidelines

<table>
<thead>
<tr>
<th>Score Point</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 points</td>
<td>The response includes the following correct Statement 1 with a correct Justification:</td>
</tr>
<tr>
<td></td>
<td><strong>Statement 1:</strong></td>
</tr>
<tr>
<td></td>
<td>a) No, the events are independent.</td>
</tr>
<tr>
<td></td>
<td><strong>Justification:</strong></td>
</tr>
<tr>
<td></td>
<td>a) The probability of the student arriving late given that he or she goes to bed by 10:00 p.m. ( \frac{8}{80} ) is equal to the probability that the student arrives late given that he or she goes to bed after 10:00 p.m. ( \frac{1}{10} ), so the two events are independent of each other.</td>
</tr>
<tr>
<td></td>
<td>OR</td>
</tr>
<tr>
<td></td>
<td>b) The probability of going to bed by 10:00 p.m. is ( \frac{80}{90} ). The probability of arriving to school on time is ( \frac{81}{90} ). Since the probability of doing both is ( \frac{72}{90} = \frac{80}{90} \times \frac{81}{90} ), the two events are independent.</td>
</tr>
<tr>
<td></td>
<td>OR</td>
</tr>
<tr>
<td></td>
<td>c) Given that the student went to bed by 10:00 p.m., the ratio of the number of occurrences of the student arriving at the school late to the number of occurrences that the student arrives at school on time is 8 to 72. Given that student went to bed after 10:00 p.m., the ratio of the number of occurrences of the student arriving at school late to the number of occurrences of student arriving at school on time is 1 to 9. Since 8 to 72 is equivalent to 1 to 9, the events are independent.</td>
</tr>
<tr>
<td>1 point</td>
<td>The response includes the correct Statement 1 listed above with a partially correct Justification.</td>
</tr>
<tr>
<td>0 points</td>
<td>The response does not meet the criteria required to earn one point. The response indicates inadequate or no understanding of the task and/or the idea or concept needed to answer the item. It may only repeat information given in the test item. The response may provide an incorrect solution/response and the provided supportive information may be irrelevant to the item, or possibly, no other information is shown. The student may have written on a different topic or written, “I don’t know.”</td>
</tr>
</tbody>
</table>
Sample Response: 2 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

No, it doesn't because based on the information provided, the probability that the child will show up when he or she goes to bed before 10:00 is 90%. But if the student doesn't go to bed at 10:00, they still arrive at school on time 90%, meaning that the time that this student goes to bed doesn't have any affect on their attendance.
Notes on Scoring

This response earns full credit (2 points) because it shows the correct statement ("No, it doesn’t...") and the correct justification for the statement.

The two events are independent if their probabilities are equal or if the probability of them occurring together is the product of their individual probabilities.

The probability that the student arrives late given that he or she goes to bed by 10:00 p.m. is \( \frac{8}{(72+8)} \) or \( \frac{8}{80} \) or \( \frac{1}{10} \). The probability that the student arrives late given that he or she goes to bed after 10:00 p.m. is \( \frac{1}{(9+1)} \) or \( \frac{1}{10} \). Since two probabilities are equal, \( \frac{8}{80} = \frac{1}{10} \), the two events are independent.

Similarly, the probability that the student arrives on time given that he or she goes to bed by 10:00 p.m. is \( \frac{72}{80} \) or 90%. The probability that the student arrives on time given that he or she goes to bed after 10:00 p.m. is \( \frac{9}{10} \) or 90%. Since two probabilities are equal, the events are independent.

Also, the probability of going to bed by 10:00 pm is \( \frac{(72+8)}{90} \) or \( \frac{80}{90} \). The probability of arriving to school on time is \( \frac{(72+9)}{90} \) or \( \frac{81}{90} \). According to the table, the probability of going to bed and arriving at school on time is approximately \( \frac{72}{90} \). Since \( \frac{(72+9)}{90} \cdot \frac{81}{90} \) equals to \( \frac{72}{90} \), the two events are independent.

The ratio of the number of times the student arrives late to the number of times he or she arrives on time, given that the student goes to bed by 10:00 p.m., is \( \frac{8}{72} \). The ratio of the number of times the student arrives late to the number of times he or she arrives on time, given that the student goes to bed after 10:00 p.m., is \( \frac{1}{9} \). Since both ratios are equal and \( \frac{8}{72} = \frac{1}{9} \), the two events are independent.
Sample Response: 2 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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</tr>
</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

No, arriving at school on time is not affected by what time the student goes to bed. When he goes to bed on time he was still late 1/10 of the time, the same chances of being late when he did go to bed after 10.

Notes on Scoring

This response earns full credit (2 points) because it shows the correct statement (“No, arriving at school on time is not affected by what time the student goes to bed.”) and the correct justification for the statement.

The two events are independent if their probabilities are equal or if the probability of them occurring together is the product of their probabilities.

The probability that the student arrives late given that he or she goes to bed by 10:00 p.m. is \( \frac{8}{72+8} \) or \( \frac{8}{80} \) or \( \frac{1}{10} \). The probability that the student arrives late given that he goes to bed after 10:00 p.m. is \( \frac{1}{9+1} \) or \( \frac{1}{10} \). Since the two probabilities are equal, \( \frac{8}{80} = \frac{1}{10} \), the two events are independent.
Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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</tr>
</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

No because once every nine times is very similar to 8 times every 72 times

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement (“No because…”) but provides an incomplete justification for the statement.
Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

No it does not, it is the exact same chance that they will end up late to school

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement (“No it does not...”) but provides an incomplete justification for the statement.
Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

No the student arriving on time does not depend on whether or not the student goes to bed by 10 p.m. The student arrived at school late just as many times as when he went to bed at 10 p.m.

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement (“No...”) but provides an incorrect justification for the statement. The student may have confused the number of occurrences of the two events with the ratio of the events.
Sample Response: 1 point

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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</tr>
</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

It does not depend on if the student goes to bed at 10 p.m. or not. In the data shown, the student does get to school on time when he goes to bed before 10 p.m., but she also gets to school on time when he goes to bed after that time. Also, when the student goes to bed after 10 p.m., it seems that he was not late to school as much if he went to bed at 10 p.m. Therefore, it does not matter on what time the student gets to bed.

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct statement ("It does not depend on if the student goes to bed at 10 p.m. or not.") but provides a statistically incorrect justification for the statement.
Sample Response: 0 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

If the student goes to bed by 10, then the probability of him getting to school on time is much greater than him going to bed after 10. I know this because when the student went to bed before 10, the table tells me he arrived at school on time 72 times. But when he didn't go to bed until after 10, he only arrived to school early 9 times. The probability of him making it to school on time is higher if he goes to bed lot sooner.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect statement and provides a statistically incorrect justification for the situation.
Sample Response: 0 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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<td>1</td>
</tr>
</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Yes and no because everyone is different sometimes its harder to get up in the morning then for others. But since this person is doing this survey he was only late once going to bed after 10. So in this case for this person no it does not matter.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect statement based on an incorrect justification.
Sample Response: 0 points

During a 90-day semester, a student records whether he arrives at school on time and whether he goes to bed by 10:00 p.m. the night before. The results are shown in the table.

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</tr>
</tbody>
</table>

Does the student arriving at school on time depend on whether the student goes to bed by 10:00 p.m.? Justify your reasoning.

Type your answer in the space provided.

Yes, the student arriving on time does depend on whether the student goes to bed by 10:00 p.m. because in the student's records he arrives to school on time 72 times, but is late 8 times arriving. Him not going to bed on time will be 9 times he is on time to school and 1 time he is late arriving to school. So, in this situation he should go to bed around 9, then will be on time, and not be late.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect statement based on incorrect justification.
Integrated Math II
Spring 2017 Item Release

Question 8

Question and Scoring Guidelines
An online retailer conducts a random survey of its customers. The survey shows that 80% of the customers receive their purchases within four days, and 95% of those customers are satisfied with the quality of their purchases.

What percent of all customers receive their purchases within four days and are not satisfied with the quality of their purchases?

<table>
<thead>
<tr>
<th>Option</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4%</td>
</tr>
<tr>
<td>B</td>
<td>5%</td>
</tr>
<tr>
<td>C</td>
<td>19%</td>
</tr>
<tr>
<td>D</td>
<td>24%</td>
</tr>
</tbody>
</table>

**Points Possible:** 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Content Standard:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. *(S.CP.6)*
Scoring Guidelines

Rationale for Option A: **Key** – The student correctly concluded that 5% of the customers who received their purchases within four days were not satisfied (100 – 95 = 5%), and 5% of 80% of all customers who received their purchases within four days is 0.05•80 = 4%.

Rationale for Option B: This is incorrect. The student may have found that only 5% of the customers who received their purchases within four days were not satisfied, and did not multiply 5% by 80% to find the percentage of all customers.

Rationale for Option C: This is incorrect. The student may have reversed the conditional probability, found the complement of the customers who received their purchases within four days is 20% and then found 95% of 20% as 0.95•0.20 = 0.19 or 19%.

Rationale for Option D: This is incorrect. The student may have found the percentage of all customers who received their purchases within four days and were satisfied with the purchases by multiplying 95% by 80%, or 0.95•0.80 = 0.76 or 76%, and then subtracted that from 100% to get 24%.

Sample Response: 1 point

An online retailer conducts a random survey of its customers. The survey shows that 80% of the customers receive their purchases within four days, and 95% of those customers are satisfied with the quality of their purchases.

What percent of all customers receive their purchases within four days and are not satisfied with the quality of their purchases?

- 4%
- 5%
- 19%
- 24%
Integrated Math II
Spring 2017 Item Release

Question 9

Question and Scoring Guidelines
Josh has a bag containing pieces of candy. The bag contains 10 red circular pieces, 10 red square pieces, 10 blue triangular pieces, and 10 blue star-shaped pieces. He draws a red piece of candy from the bag.

What is the complement of this event?

A. He draws a blue piece.
B. He draws a square piece.
C. He draws a circular piece.
D. He draws a star-shaped piece.

Points Possible: 1

Content Cluster: Understand independence and conditional probability, and use them to interpret data.

Content Standard: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (S.CP.1)
Scoring Guidelines

Rationale for Option A: **Key** – The student correctly identified that the complement of drawing a red piece is drawing a blue piece since all of the candy can be described as either being red or blue.

Rationale for Option B: This is incorrect. The student may have incorrectly thought complement means to describe the candy using a secondary feature and chose one of the shapes that describes some of the red candies.

Rationale for Option C: This is incorrect. The student may have incorrectly thought complement means to describe the candy using a secondary feature and chose one of the shapes that describes some of the red candies.

Rationale for Option D: This is incorrect. The student may have correctly realized that the complement of drawing a red candy has to be a descriptor that cannot describe any red candy, but the student incorrectly chose the shape of a blue candy. The star-shaped candy is not the complement. It is only a part of all the blue candies, and therefore it is only one part of the complement instead of the entire complement.

Sample Response: 1 point

Josh has a bag containing pieces of candy. The bag contains 10 red circular pieces, 10 red square pieces, 10 blue triangular pieces, and 10 blue star-shaped pieces. He draws a red piece of candy from the bag.

What is the complement of this event?

- He draws a blue piece.
- He draws a square piece.
- He draws a circular piece.
- He draws a star-shaped piece.
Integrated Math II
Spring 2017 Item Release

Question 10

Question and Scoring Guidelines
**Question 10**

Two pairs of parallel lines intersect to form a parallelogram as shown.

![Diagram of parallelogram](image)

Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m \parallel n ) ( k \parallel l )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

\( \angle 1 \cong \angle 2 \)  
\( \angle 1 \cong \angle 3 \)  
\( \angle 2 \cong \angle 3 \)  
\( \angle 1 \cong \angle 1 \)  

Alternate exterior angles are congruent.  
Alternate interior angles are congruent.  
Transitive property of congruence.  
Opposite angles are congruent.  
Corresponding angles are congruent.

**Points Possible:** 1

**Content Cluster:** Prove geometric theorems.

**Content Standard:** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G.CO.11)
Scoring Guidelines

Exemplar Response

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m \parallel n$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$k \parallel l$</td>
<td></td>
</tr>
<tr>
<td>2. $\angle 1 \equiv \angle 2$</td>
<td>2. Alternate interior angles are congruent.</td>
</tr>
<tr>
<td>3. $\angle 2 \equiv \angle 3$</td>
<td>3. Corresponding angles are congruent.</td>
</tr>
<tr>
<td>4. $\angle 1 \equiv \angle 3$</td>
<td>4. Transitive property of congruence</td>
</tr>
</tbody>
</table>

Other Correct Responses

- Line 2 and Line 3 can be switched.
- The sentence “Corresponding angles are congruent.” may be added to Reason 4 without penalty, or be acceptable as the only response in Statement 4.

For this item, a full-credit response includes:

- A correct proof (1 point).
Integrated Math II
Spring 2017 Item Release

Question 10

Sample Responses
Sample Response: 1 point

Two pairs of parallel lines intersect to form a parallelogram as shown.

Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>1. ( m \parallel n ) ( k \parallel l )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \equiv \angle 2 )</td>
<td>2. Alternate interior angles are congruent.</td>
</tr>
<tr>
<td>3. ( \angle 2 \equiv \angle 3 )</td>
<td>3. Corresponding angles are congruent.</td>
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<td>4. ( \angle 1 \equiv \angle 3 )</td>
<td>4. Transitive property of congruence</td>
</tr>
</tbody>
</table>

Alternate exterior angles are congruent.

\( \angle 1 \equiv \angle 1 \)

Opposite angles are congruent.
Notes on Scoring

This response earns full credit (1 point) because it correctly completes the proof to show that opposite angles in parallelograms are congruent.

Following from the given information that the pairs of opposite sides in the parallelogram are parallel, there are two pairs of congruent angles formed by parallel lines and a transversal line. For example, if $m \parallel n$, then $\angle 1 \cong \angle 2$ by the Alternate Interior Angles Theorem because the angles are formed by two parallel lines $m$ and $n$ and the transversal line $k$ (step 2). Likewise, if $k \parallel l$, then $\angle 2 \cong \angle 3$ by the Corresponding Angles Theorem because the angles are formed by two parallel lines $k$ and $l$ and the transversal line $m$ (step 3). The proof would also be correct if steps 2 and 3 are switched. Lastly, if $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$ by the Transitive Property of Congruence (step 4).
Sample Response: 1 point

Two pairs of parallel lines intersect to form a parallelogram as shown.

![Diagram of parallelogram with parallel lines and angles labeled 1, 2, 3, and m, n, k, l.]

Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
</table>
| 1. \( m \parallel n \)  
   \( k \parallel l \) | 1. Given                        |
| 2. \( \angle 2 \equiv \angle 3 \) | 2. Corresponding angles are congruent. |
| 3. \( \angle 1 \equiv \angle 2 \) | 3. Alternate interior angles are congruent. |
| 4. \( \angle 1 \equiv \angle 3 \) | 4. Transitive property of congruence |

Alternate exterior angles are congruent.

\( \angle 1 \equiv \angle 1 \)  

Opposite angles are congruent.
Notes on Scoring

This response earns full credit (1 point) because it correctly completes the proof to show that opposite angles in parallelograms are congruent (with steps 2 and 3 switched).

Following from the given information that the pairs of opposite sides in the parallelogram are parallel, there are two pairs of congruent angles formed by parallel lines and a transversal line. For example, if \(k\parallel l\), then \(\angle 2 \cong \angle 3\) by the Corresponding Angle Theorem because the angles are formed by two parallel lines \(k\) and \(l\) and the transversal line \(m\) (step 2). Likewise, if \(m\parallel n\), then \(\angle 1 \cong \angle 2\) by the Alternate Interior Angle Theorem because the angles are formed by two parallel lines \(m\) and \(n\) and the transversal line \(k\) (step 3). Lastly, if \(\angle 1 \cong \angle 2\) and \(\angle 2 \cong \angle 3\), then \(\angle 1 \cong \angle 3\) by the Transitive Property of Congruence (step 4).
Sample Response: 0 points

Two pairs of parallel lines intersect to form a parallelogram as shown.

Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
</table>
| 1. $m \parallel n$  
    $k \parallel l$ | 1. Given                                    |
| 2. $\angle 2 \equiv \angle 3$ | 2. Corresponding angles are congruent.    |
| 3. $\angle 1 \equiv \angle 2$ | 3. Alternate exterior angles are congruent.| |
| 4. $\angle 1 \equiv \angle 3$ | 4. Transitive property of congruence       |

Alternate interior angles are congruent.
Opposite angles are congruent.

$\angle 1 \equiv \angle 1$

Notes on Scoring

This response earns no credit (0 points) because it shows the incorrect reasoning in step 3 for the proof that opposite angles in parallelograms are congruent.

The student may have confused alternate interior angles with alternate exterior angles.
Sample Response: 0 points

Two pairs of parallel lines intersect to form a parallelogram as shown.

Place statements and reasons in the table to complete the proof that the opposite angles of a parallelogram are congruent.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
</table>
| 1. \( m \parallel n \)
\( k \parallel l \) | 1. Given                                     |
| 2. \( \angle 1 \neq \angle 2 \)     | 2.                                           |
| 3. \( \angle 2 \neq \angle 3 \)     | 3.                                           |
| 4. \( \angle 1 \neq \angle 3 \)     | 4.                                           |

\( \angle 1 \neq \angle 1 \)

Alternate exterior angles are congruent.
Alternate interior angles are congruent.
Transitive property of congruence
Opposite angles are congruent.
Corresponding angles are congruent.

Notes on Scoring

This response earns no credit (0 points) because it incorrectly completes the proof to show that opposite angles in parallelograms are congruent. Although the proof shows all correct statements, it misses all corresponding reasons for each of the three steps.
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Question 11

Question and Scoring Guidelines
Angle A is the complement of angle B.

Which equation about the two angles must be true?

A. \( \sin A = \sin B \)
B. \( \sin A = \cos A \)
C. \( \cos B = \sin B \)
D. \( \cos A = \sin B \)

**Points Possible:** 1

**Content Cluster:** Define trigonometric ratios, and solve problems involving right triangles.

**Content Standard:** Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have confused complementary angles with congruent angles and concluded that the sine of two congruent angles are equal.

Rationale for Option B: This is incorrect. The student may have recognized that the relationship involves the sine of an angle equaling the cosine of an angle but did not realize that it is the sine of one angle being equal to the cosine of its complement.

Rationale for Option C: This is incorrect. The student may have recognized that the relationship involves the sine of an angle equaling the cosine of an angle but did not realize that it is the sine of one angle being equal to the cosine of its complement.

Rationale for Option D: Key – The student correctly noted that the sine of an angle is equal to the cosine of its complement.

Sample Response: 1 point

Angle A is the complement of angle B.

Which equation about the two angles must be true?

A $\sin A = \sin B$

B $\sin A = \cos A$

C $\cos B = \sin B$

D $\cos A = \sin B$
Integrated Math II
Spring 2017 Item Release

Question 12

Question and Scoring Guidelines
Question 12

James correctly proves the similarity of triangles DAC and DBA as shown.

His incomplete proof is shown.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m\angle CAB = m\angle ADB = 90^\circ$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle ADB + m\angle ADC = 180^\circ$</td>
<td>2. Angles in a linear pair are supplementary.</td>
</tr>
<tr>
<td>3. $90^\circ + m\angle ADC = 180^\circ$</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. $m\angle ADC = 90^\circ$</td>
<td>4. Subtraction property of equality</td>
</tr>
<tr>
<td>5. $\angle CAB \cong \angle ADB$</td>
<td>5. Definition of congruent angles</td>
</tr>
<tr>
<td>6. $\angle CAB \cong \angle ADB$</td>
<td>6. Reflexive property of congruence</td>
</tr>
<tr>
<td>7. $\triangle ABC \sim \triangle DBA$</td>
<td>7. ?</td>
</tr>
<tr>
<td>8. $\triangle ABC \sim \triangle DAC$</td>
<td>8. Substitution</td>
</tr>
<tr>
<td>9. $\triangle DBA \sim \triangle DAC$</td>
<td></td>
</tr>
</tbody>
</table>

What is the missing reason for the seventh statement?

A. CPCTC
B. AA postulate
C. All right triangles are similar.
D. Transitive property of similarity

Points Possible: 1

Content Cluster: Prove theorems involving similarity.

Content Standard: Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have not recognized that CPCTC is the reasoning that follows from congruence, not similarity, and that it has not been proven that the two triangles are congruent.

Rationale for Option B: Key – The student correctly noticed that James' proof shows that two pairs of corresponding angles are congruent and that this satisfies the AA criterion in showing that two triangles are similar.

Rationale for Option C: This is incorrect. The student may have seen that James has proven that the two right triangles in the graphic are similar, but incorrectly made the generalization that since these two right triangles are similar, all right triangles are similar.

Rationale for Option D: This is incorrect. The student may have chosen the transitive property because the proof shows two similarity statements and one more similarity statement follows after that.
James correctly proves the similarity of triangles DAC and DBA as shown.

His incomplete proof is shown.

<table>
<thead>
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<td>6. Reflexive property of congruence</td>
</tr>
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<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>$\triangle ABC \sim \triangle DAC$</td>
<td></td>
</tr>
<tr>
<td>8. $\triangle DBA \sim \triangle DAC$</td>
<td>8. Substitution</td>
</tr>
</tbody>
</table>

What is the missing reason for the seventh statement?

- (A) CPCTC
- (B) AA postulate
- (C) All right triangles are similar.
- (D) Transitive property of similarity
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Question 13

Question and Scoring Guidelines
Question 13

Points Possible: 1

Content Cluster: Define trigonometric ratios, and solve problems involving right triangles.

Content Standard: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)
Scoring Guidelines
Exemplar Response

- 2.4

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).
Sample Response: 1 point

Notes on Scoring

This response earns full credit (1 point) because it shows the correct \( \tan(A) \). In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. Based on this definition, \( \tan(A) = \frac{CB}{AB} = \frac{24}{10} \) or \( 
\frac{12}{5} \) or 2.4.
Sample Response: 1 point

Notes on Scoring

This response earns full credit (1 point) because it shows the correct \( \tan(A) \). In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. Based on this definition, \( \tan(A) = \frac{CB}{AB} = \frac{24}{10} \).
Sample Response: 0 points

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect $\tan(A)$. In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. In this situation, the student may have found the reciprocal of the tangent, or the ratio of the lengths of the adjacent leg to the opposite leg.
Sample Response: 0 points

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect tan(A). In the right triangles, the tangent of the acute angle is the ratio of the lengths of the opposite leg to the adjacent leg. In this situation, the student may have found the sin(A), or the ratio of the lengths of the opposite leg to the hypotenuse.
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Question 14

Question and Scoring Guidelines
Circle B is centered at the origin and has a radius of $r$. Circle A is centered at $(r, 0)$ and is tangent to circle B as shown.

Which sequence of transformations can be performed on circle A to show that it is similar to circle B?

- A: a translation $r$ units to the right and then a dilation, centered at the origin, by a scale factor of 2
- B: a translation $r$ units to the left and then a dilation, centered at the origin, by a scale factor of $\frac{1}{2}$
- C: a translation $r$ units to the left and then a dilation, centered at the origin, by a scale factor of 4
- D: a translation $r$ units to the right and then a dilation, centered at the origin, by a scale factor of $\frac{1}{2}$

Points Possible: 1

Content Cluster: Understand and apply theorems about circles.

Content Standard: Prove that all circles are similar. (G.C.1)
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have identified a sequence of transformations that could be performed on circle B to show that it is similar to circle A instead of the reverse.

Rationale for Option B: Key – The student correctly identified that translating circle A to the left \( r \) units would make the centers coincide and then dilating circle A by a scale factor of \( \frac{1}{2} \) would reduce the size of circle A to the size of circle B to show that the circles are similar.

Rationale for Option C: This is incorrect. The student understood that the circle A needed to be translated to the left \( r \) units, but the student may have thought the scale factor applied to circle A would be 4 since the radius of circle B is \( \frac{1}{4} \) of the diameter of circle A, but a dilation by the scale factor 4 would make circle A even larger.

Rationale for Option D: This is incorrect. The student may have applied each transformation to a different circle—the translation of circle B to the right followed by the dilation of circle A by a scale factor of \( \frac{1}{2} \).

Sample Response: 1 point

Circle B is centered at the origin and has a radius of \( r \). Circle A is centered at \((r, 0)\) and is tangent to circle B as shown.

Which sequence of transformations can be performed on circle A to show that it is similar to circle B?

- \( \text{A} \) a translation \( r \) units to the right and then a dilation, centered at the origin, by a scale factor of 2
- \( \text{B} \) a translation \( r \) units to the left and then a dilation, centered at the origin, by a scale factor of \( \frac{1}{2} \)
- \( \text{C} \) a translation \( r \) units to the left and then a dilation, centered at the origin, by a scale factor of 4
- \( \text{D} \) a translation \( r \) units to the right and then a dilation, centered at the origin, by a scale factor of \( \frac{1}{2} \)
Integrated Math II
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Question 15

Question and Scoring Guidelines
Question 15

Two functions with different vertices are shown.

\[ g(x) = -x^2 + 6x + 2 \]

How many times larger is the \( y \)-value of the higher vertex than the \( y \)-value of the lower vertex?

Points Possible: 1

Content Cluster: Analyze functions using different representations.

Content Standard: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F.IF.9)
Scoring Guidelines
Exemplar Response

- 2

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).
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Spring 2017 Item Release

Question 15

Sample Responses
Sample Response: 1 point

Notes on Scoring

This response earns full credit (1 point) because it correctly compares the y-values of the vertices of two quadratic functions represented in different forms.

The vertex of the function $f(x)$, which is shown on the graph, is $(0.5, 5.5)$ where the y-value is 5.5. The vertex of the function $g(x)$ can be found using a variety of methods. One method is to use a graphing utility to graph $g(x) = -x^2 + 6x + 2$, and then use the graph to find the vertex, thereby finding that the vertex of $g(x)$ is $(3, 11)$ where the y-value is 11. Since 11 is the larger of the y-values and 5.5 is the smaller of the y-values, 11 is 2 times larger than 5.5 because $\frac{11}{5.5} = 2$. 
Sample Response: 1 point

Two functions with different vertices are shown.

\[ g(x) = -x^2 + 6x + 2 \]

How many times larger is the y-value of the higher vertex than the y-value of the lower vertex?

\[ \frac{11}{5.5} \]

Notes on Scoring

This response earns full credit (1 point) because it correctly compares the y-values of the vertices of two quadratic functions represented in different forms.

The vertex of the function \( f(x) \), which is shown on the graph, is (0.5, 5.5) where the y-value is 5.5. The vertex of the function \( g(x) \) can be found using a variety of methods. One method is to use a graphing utility to graph \( g(x) = -x^2 + 6x + 2 \), and then use the graph to find the vertex, thereby finding that the vertex of \( g(x) \) is (3, 11) where the y-value is 11.

Since 11 is the larger of the y-values and 5.5 is the smaller of the y-values, \( \frac{11}{5.5} \) represents how many times larger one y-value is than the other.
Sample Response: 0 points

How many times larger is the y-value of the higher vertex than the y-value of the lower vertex?

Notes on Scoring

This response earns no credit (0 points) because it incorrectly compares the y-values of vertices of two quadratic functions represented in different ways. The student may have calculated the y-value of the vertex of function $g(x)$ and did not compare it to the y-value of the vertex of function $f(x)$. 
Notes on Scoring

This response earns no credit (0 points) because it incorrectly compares the \(y\)-values of vertices of two quadratic functions represented in different ways. The student may have identified the \(y\)-value of the vertex of function \(f(x)\) and did not compare it to the \(y\)-value of the vertex of function \(g(x)\).
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Question 16

Question and Scoring Guidelines
Question 16

A radioactive substance decays exponentially. The initial amount of the substance is 3 grams, and over time, \( t \), the amount, \( S(t) \), in grams, approaches 0.

Which graph could be a model for \( f(t) \)?

- [A] \( f(t) \)
- [B] \( f(t) \)
- [C] \( f(t) \)
- [D] \( f(t) \)

**Points Possible:** 1

**Content Cluster:** Interpret functions that arise in applications in terms of the context.

**Content Standard:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)
Scoring Guidelines

**Rationale for Option A:** This is incorrect. The student may have confused an exponential decay function with an exponential growth function that increases instead of decreases. He or she may have also incorrectly thought that the initial amount of the substance is the amount the substance approaches over time.

**Rationale for Option B:** This is incorrect. The student may have correctly understood that an exponentially decaying function decreases, but incorrectly thought that the initial amount of the substance is the amount the substance approaches over time.

**Rationale for Option C:** Key – The student correctly understood that the initial amount of 3 is represented by the y-intercept, (0, 3), and that the graph of an exponential decay function decreases as it approaches the x-axis.

**Rationale for Option D:** This is incorrect. The student may have understood that the initial amount of 3 is represented by the y-intercept (0, 3), but the student may have confused a growth function with a decay function and thought that the decaying function increases and also incorrectly concluded that the graph approaches an x-value of 0 going from right to left.
A radioactive substance decays exponentially. The initial amount of the substance is 3 grams, and over time, t, the amount, S(t), in grams, approaches 0.

Which graph could be a model for f(t)?

Sample Response: 1 point
Integrated Math II
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Question 17

Question and Scoring Guidelines
Question 17

Which function is equivalent to \( f(x) = (x + 3)(x - 9)? \)

A. \( f(x) = (x + 3)^2 - 9 \)
B. \( f(x) = (x + 3)^2 - 18 \)
C. \( f(x) = (x - 3)^2 - 27 \)
D. \( f(x) = (x - 3)^2 - 36 \)

Points Possible: 1

Content Cluster: Analyze functions using different representations.

Content Standard: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. \( (F.IF.8) \)
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have selected the function with the same numerals as the original, thinking these are equivalent forms of \( f(x) \).

Rationale for Option B: This is incorrect. The student may have incorrectly expanded the original expression to get \( f(x) = x^2 + 6x - 27 \), then completed the square to get \( f(x) = (x^2 + 6x + 9) - 27 + 9 \), but added the 9 outside the parentheses instead of subtracting the 9 to maintain equivalency with the 9 inside of the parentheses. Then he or she may have changed the expression to \( f(x) = (x + 3)^2 - 18 \).

Rationale for Option C: This is incorrect. The student may have correctly expanded the original expression, and then completed the square to get \( f(x) = (x^2 - 6x + 9) - 27 \), but forgot to subtract 9 from outside the parenthesis to maintain equivalency when completing the square.

Rationale for Option D: Key – The student correctly expanded the original expression to get \( f(x) = x^2 - 6x - 27 \) and then completed the square to get \( f(x) = (x^2 - 6x + 9) - 27 - 9 \), which is equivalent to \( f(x) = (x - 3)^2 - 36 \).

Sample Response: 1 point

Which function is equivalent to \( f(x) = (x + 3)(x - 9) \)?

A. \( f(x) = (x + 3)^2 - 9 \)
B. \( f(x) = (x + 3)^2 - 18 \)
C. \( f(x) = (x - 3)^2 - 27 \)
D. \( f(x) = (x - 3)^2 - 36 \)
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Question 18

Question and Scoring Guidelines
Question 18

Angle \( \theta \) is in the first quadrant and \( \cos \theta = \frac{4}{5} \).

What is the value of \( \sin \theta \)?

Points Possible: 1

Content Cluster: Prove and apply trigonometric identities.

Content Standard: Prove the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \), and use it to find \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \) given \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \) and the quadrant of the angle. (F.TF.8)
Scoring Guidelines
Exemplar Response

- $\frac{3}{5}$

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).
Integrated Math II
Spring 2017 Item Release

Question 18

Sample Responses
Sample Response: 1 point

Angle $\theta$ is in the first quadrant and $\cos \theta = \frac{4}{5}$.

What is the value of $\sin \theta$?

\[
\frac{3}{5}
\]
Notes on Scoring

This response earns full credit (1 point) because it shows the correct value for sine of the angle θ in the first quadrant. The solution is based on associating angle θ with a right triangle that is placed in the first quadrant of the rectangular coordinate system in such a way that the vertex of the angle θ coincides with the origin, one leg runs along the x-axis and another leg is perpendicular to the x-axis, or parallel to the y-axis.

Following the definition, in a right triangle, cosine θ is a ratio of the adjacent leg’s length to the hypotenuse’s length, and since \( \cos \theta = \frac{4}{5} \), the length of the adjacent leg is 4 and the length of the hypotenuse is 5 units. By applying the Pythagorean Theorem, \( (4^2 + \text{leg}^2 = 5^2) \), length of the unknown opposite leg is 3 units. Following the definition in a right triangle, sine θ is a ratio of the opposite leg’s length to the hypotenuse’s length, or \( \sin \theta = \frac{3}{5} \).
Sample Response: 1 point

Angle $\theta$ is in the first quadrant and $\cos \theta = \frac{4}{5}$.

What is the value of $\sin \theta$?

0.6
Notes on Scoring

This response earns full credit (1 point) because it shows the correct value for sine of the angle $\theta$ in the first quadrant. The solution is based on associating angle $\theta$ with a right triangle that is placed in the first quadrant of the rectangular coordinate system in such a way that the vertex of the angle $\theta$ coincides with the origin, one leg runs along the $x$-axis and another leg is perpendicular to the $x$-axis, or parallel to the $y$-axis.

Following the definition, in a right triangle, cosine $\theta$ is a ratio of adjacent leg’s length to the hypotenuse’s length, and since $\cos \theta = \frac{4}{5}$, the length of the adjacent leg is 4 and the length of the hypotenuse is 5 units. By applying the Pythagorean Theorem ($4^2 + \text{leg}^2 = 5^2$), the length of the unknown opposite leg is 3 units. Following the definition, in a right triangle, sine $\theta$ is the ratio of the opposite leg’s length to the hypotenuse’s length, or $\sin \theta = \frac{3}{5}$, or 0.6.
Sample Response: 0 points

Angle $\theta$ is in the first quadrant and $\cos \theta = \frac{4}{5}$.

What is the value of $\sin \theta$?

\[
\begin{array}{c}
\frac{3}{4}
\end{array}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for sine of the angle $\theta$ in the first quadrant. The student may have incorrectly used the ratio for the tangent, such as tangent $\theta = \frac{\text{opposite leg's length}}{\text{adjacent leg's length}}$, or $\frac{3}{4}$, instead of sine $\theta = \frac{\text{opposite leg's length}}{\text{hypotenuse's length}}$, or $\frac{3}{5}$. 
Sample Response: 0 points

Angle $\theta$ is in the first quadrant and $\cos \theta = \frac{4}{5}$.

What is the value of $\sin \theta$?

\[
\frac{5}{3}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for sine of the angle $\theta$ in the first quadrant. The student may have used an incorrect ratio for sine $\theta$, such as hypotenuse’s length/opposite leg’s length or $\frac{5}{3}$, instead of opposite leg’s length/hypotenuse’s length, or $\frac{3}{5}$. 
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Question 19

Question and Scoring Guidelines
The graph of \( f(x) = x^2 \) is transformed to create the graph of \( g(x) = f(x) + 3 \).

Which coordinate plane shows the graph of \( g(x) \)?

Points Possible: 1

Content Cluster: Build new functions from existing functions.

Content Standard: Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k, kf(x), f(kx), \) and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (F.BF.3)
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may have thought that $f(x) + k$, where $k$ is a positive number, representing a horizontal shift of $f(x)$ 3 units to the right.

Rationale for Option B: Key – The student recognizes that $f(x)+3$ represents a vertical shift of $f(x)$ 3 units up.

Rationale for Option C: This is incorrect. The student may have thought that $f(x) + k$, where $k$ is a positive number, representing a horizontal shift of $f(x)$ 3 units to the left.

Rationale for Option D: This is incorrect. The student may have thought that $f(x) + k$, where $k$ is a positive number, representing a vertical shift of $f(x)$ 3 units down.
The graph of $f(x) = x^2$ is transformed to create the graph of $g(x) = f(x) + 3$.

Which coordinate plane shows the graph of $g(x)$?
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Spring 2017 Item Release

Question 20

Question and Scoring Guidelines
Question 20

A function is shown.

\[ f(x) = x^2 - 3x + 3 \]

What is the average rate of change of the function over the interval \( 1 \leq x \leq 4 \)?

Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context.

Content Standard: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. (F.IF.6)
Scoring Guidelines
Exemplar Response

- 2

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- The correct value (1 point).
Integrated Math II
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Question 20

Sample Responses
Sample Response: 1 point

A function is shown.
\( f(x) = x^2 - 3x + 3 \)

What is the average rate of change of the function over the interval \( 1 \leq x \leq 4 \)?

2

Notes on Scoring

This response earns full credit (1 point) because it correctly calculates the average rate of change of a function over a specified interval.

The average rate of change of a function, \( f(x) \), over the closed interval \( a \leq x \leq b \) is given by \( \frac{f(b)-f(a)}{b-a} \). In this situation, the average rate of change over the interval \( 1 \leq x \leq 4 \) is \( \frac{f(4)-f(1)}{4-1} \). To find \( f(4) \), substitute 4 for \( x \), so \( f(4) = 4^2 - 3(4) + 3 \) which results in \( f(4) = 7 \). To find \( f(1) \), substitute 1 for \( x \), so \( f(1) = 1^2 - 3(1) + 3 \) which results in \( f(1) = 1 \).

Therefore, \( \frac{f(4)-f(1)}{4-1} = \frac{7-1}{4-1} = \frac{6}{3} = 2 \).
Sample Response: 1 point

A function is shown.
\[ f(x) = x^2 - 3x + 3 \]

What is the average rate of change of the function over the interval \( 1 \leq x \leq 4 \)?

\[
\frac{f(4) - f(1)}{4 - 1} = \frac{6}{3}.
\]

Notes on Scoring

This response earns full credit (1 point) because it correctly calculates the average rate of change of a function over a specified interval.

The average rate of change of a function, \( f(x) \), over the closed interval \( a \leq x \leq b \) is given by \( \frac{f(b) - f(a)}{b - a} \). In this situation, the average rate of change over the interval \( 1 \leq x \leq 4 \) is \( \frac{f(4) - f(1)}{4-1} \). To find \( f(4) \), substitute 4 for \( x \), so \( f(4) = 4^2 - 3(4) + 3 \) which results in \( f(4) = 7 \). To find \( f(1) \), substitute 1 for \( x \), so \( f(1) = 1^2 - 3(1) + 3 \) which results in \( f(1) = 1 \). Therefore, \( \frac{f(4) - f(1)}{4-1} = \frac{7-1}{3} = \frac{6}{3} \). The average rate of change in the non-lowest terms also earns full credit.
Sample Response: 0 points

A function is shown.

\[ f(x) = x^2 - 3x + 3 \]

What is the average rate of change of the function over the interval \(1 \leq x \leq 4\)?

\[
\frac{6}{4}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly calculated average rate of change of a function over a specified interval.

The average rate of change of a function, \( f(x) \), over the closed interval \(a \leq x \leq b\) is given by \( \frac{f(b) - f(a)}{b - a} \). The student may have calculated \( f(b) - f(a) \) correctly, but miscalculated \( b - a \) incorrectly to get \( \frac{6}{4} \).
Sample Response: 0 points

A function is shown.

\[ f(x) = x^2 - 3x + 3 \]

What is the average rate of change of the function over the interval \(1 \leq x \leq 4\)?

\[ \frac{\frac{1}{2}}{} \]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly calculated average rate of change of a function over a specified interval. The student may have used an incorrect formula, such as \( \frac{b-a}{f(b)-f(a)} \) instead of by \( \frac{b-a}{f(b)-f(a)} \).
Integrated Math II
Spring 2017 Item Release

Question 21

Question and Scoring Guidelines
Question 21

A system of equations is shown.

\[ y + 1 = 2x \]
\[ x^2 - 4x = y - 4 \]

What are the solutions to the system of equations?

\[
(\quad, \quad) \\
(\quad, \quad)
\]

Points Possible: 2

Content Cluster: Solve systems of equations.

Content Standard: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line \( y = -3x \) and the circle \( x^2 + y^2 = 3 \). (A.REI.7)
Scoring Guidelines

Exemplar Response

- (1, 1)
- (5, 9)

Other Correct Responses

- The order of the ordered pair solutions can be switched

For this item, a full-credit response includes:

- One correct solution (1 point)
  AND
- Another correct solution (1 point).
Sample Response: 2 points

A system of equations is shown.

\[ y + 1 = 2x \]
\[ x^2 - 4x = y - 4 \]

What are the solutions to the system of equations?

\[(1, 1), (5, 9)\]

Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions (ordered pairs) to a system that consists of a linear equation and a simple quadratic equation in two variables. There are several methods to solve the system. One method is to rearrange the first equation to represent \( y \) in terms of \( x \), or \( y = 2x - 1 \), and then substitute the expression \((2x-1)\) for \( y \) into the other equation to create a quadratic equation with only one variable, or \( x^2 - 4x = (2x - 1) - 4 \). Next, change the quadratic equation into standard form, or \( x^2 - 6x + 5 = 0 \), and then factor it as \((x - 1)(x - 5) = 0\). By the Zero – Product Property, each factor in the product has to be set equal to zero, or \( x - 1 = 0 \) and \( x - 5 = 0 \). From here, \( x = 1 \) and \( x = 5 \). By substituting each \( x \)-value back into the equation \( y = 2x - 1 \), the corresponding \( y \)-values are \( y = 2(1) - 1 = 1 \) and \( y = 2(5) - 1 = 9 \). Since the solutions to the system are ordered pairs, the correct answers are \((1, 1)\) and \((5, 9)\).
Sample Response: 2 points

A system of equations is shown.

\[ y + 1 = 2x \]
\[ x^2 - 4x = y - 4 \]

What are the solutions to the system of equations?

(5, 9)  
(1, 1)

Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions (ordered pairs) to a system that consists of a linear equation and a simple quadratic equation in two variables. There are several methods to solve the system. One method is to rearrange the first equation to represent \( y \) in terms of \( x \), or \( y = 2x - 1 \), and then substitute the expression \((2x - 1)\) for \( y \) into the other equation to create a quadratic equation with only one variable, or \( x^2 - 4x = (2x - 1) - 4 \). Next, change the quadratic equation into standard form, or \( x^2 - 6x + 5 \) and then factor it as \((x - 1)(x - 5) = 0\). By the Zero – Product Property, each factor in the product has to be set equal to zero, or \( x - 1 = 0 \) and \( x - 5 = 0 \). From here, \( x = 1 \) and \( x = 5 \). By substituting each \( x \)-value back into the equation \( y = 2x - 1 \), the corresponding \( y \)-values are \( y = 2(1) - 1 = 1 \) and \( y = 2(5) - 1 = 9 \). Since the solutions to the system are ordered pairs, and the order of these ordered pairs does not matter, the response \((5, 9)\) and \((1, 1)\) is also correct.
Sample Response: 1 point

A system of equations is shown.

\[ y + 1 = 2x \]
\[ x^2 - 4x = y - 4 \]

What are the solutions to the system of equations?

(1, 1), (5, 5)

Notes on Scoring

This response earns partial credit (1 point) because it shows only one correct solution (ordered pair) to a system of a linear equation and a simple quadratic equation in two variables. The correct response for the second ordered pair is (5, 9), not (5, 5).
Sample Response: 1 point

A system of equations is shown.

\[ y + 1 = 2x \]
\[ x^2 - 4x = y - 4 \]

What are the solutions to the system of equations?

(1, 1), (6, 11)

Notes on Scoring

This response earns partial credit (1 point) because it shows only one correct solution (ordered pair) to a system of a linear equation and a simple quadratic equation in two variables. The correct response for the second ordered pair is (5, 9), not (6, 11).
Sample Response: 0 points

A system of equations is shown.

\[ y + 1 = 2x \]
\[ x^2 - 4x = y - 4 \]

What are the solutions to the system of equations?

\[
\begin{array}{ccc}
( & -1 & , & -1 \\
( & -5 & , & -9 \\
\end{array}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows no correct solutions (ordered pairs) to a system of a linear equation and a simple quadratic equation in two variables.
Sample Response: 0 points

A system of equations is shown.

\[ y + 1 = 2x \]
\[ x^2 - 4x = y - 4 \]

What are the solutions to the system of equations?

\[
\begin{array}{c}
(0, -1) \\
(4, 7)
\end{array}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows no correct solutions (ordered pairs) to a system of a linear equation and a simple quadratic equation in two variables.