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<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equation Item</td>
<td>Analyze functions using different representations.</td>
<td>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. (F.IF.7a)</td>
<td>--</td>
<td>1 point</td>
</tr>
<tr>
<td>3</td>
<td>Equation Item</td>
<td>Find arc lengths and areas of sectors of circles.</td>
<td>Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G.C.5)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>4</td>
<td>Multiple Choice</td>
<td>Create equations that describe numbers or relationships.</td>
<td>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R. (A.CED.4)</td>
<td>B</td>
<td>1 point</td>
</tr>
<tr>
<td>5</td>
<td>Equation Item</td>
<td>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</td>
<td>Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. (S.CP.6)</td>
<td>---</td>
<td>1 point</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td>6</td>
<td>Multiple Choice</td>
<td>Prove theorems involving similarity.</td>
<td>Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G.SRT.5)</td>
<td>B</td>
<td>1 point</td>
</tr>
<tr>
<td>9</td>
<td>Equation Item</td>
<td>Define trigonometric ratios and solve problems involving right triangles.</td>
<td>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>10</td>
<td>Multiple Choice</td>
<td>Analyze functions using different representations.</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F.IF.9)</td>
<td>A</td>
<td>1 point</td>
</tr>
<tr>
<td>11</td>
<td>Table Input</td>
<td>Solve systems of equations.</td>
<td>Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. (A.REI.7)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>14</td>
<td>Equation Item</td>
<td>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</td>
<td>Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. (S.CP.7)</td>
<td>---</td>
<td>1 point</td>
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<tr>
<td>20</td>
<td>Equation Item</td>
<td>Understand independence and conditional probability and use them to interpret data.</td>
<td>Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (S.CP.2)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>21</td>
<td>Multiple Select</td>
<td>Create equations that describe numbers or relationships.</td>
<td>Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A.CED.1)</td>
<td>C, D</td>
<td>1 point</td>
</tr>
<tr>
<td>23</td>
<td>Multiple Choice</td>
<td>Interpret functions that arise in applications in terms of the context.</td>
<td>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)</td>
<td>A</td>
<td>1 point</td>
</tr>
<tr>
<td>34</td>
<td>Equation Item</td>
<td>Interpret functions that arise in applications in terms of the context.</td>
<td>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)</td>
<td>---</td>
<td>2 points</td>
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#### Spring 2018 Item Release
#### Content Summary and Answer Key

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<tr>
<td>35</td>
<td>Multiple</td>
<td>Perform arithmetic operations on polynomials.</td>
<td>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1)</td>
<td>A</td>
<td>1 point</td>
</tr>
<tr>
<td>36</td>
<td>Equation Item</td>
<td>Define trigonometric ratios and solve problems involving right triangles.</td>
<td>Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>39</td>
<td>Equation Item</td>
<td>Explain volume formulas and use them to solve problems.</td>
<td>Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G.GMD.3)</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>44</td>
<td>Multiple</td>
<td>Prove theorems involving similarity.</td>
<td>Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely: the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)</td>
<td>A</td>
<td>1 point</td>
</tr>
<tr>
<td>45</td>
<td>Multiple</td>
<td>Extend the properties of exponents to rational exponents.</td>
<td>Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define ( \sqrt[3]{5} ) to be the cube root of 5 because we want ( (\sqrt[3]{5})^3 = 5 ), ( \left(\frac{1}{2}\right)^3 ) to hold, so ( \left(\frac{1}{2}\right)^3 ) must equal 5. (N.RN.1)</td>
<td>B</td>
<td>1 point</td>
</tr>
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### Integrated Math II
**Spring 2018 Item Release**
**Content Summary and Answer Key**

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</table>
| 49            | Graphic Response| Understand similarity in terms of similarity transformations. | Verify experimentally the properties of dilations given by a center and a scale factor: \((G.SRT.1)\)  
\(a\). A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | ---         | 1 point |
| 50            | Table Input     | Understand independence and conditional probability and use them to interpret data. | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. \((S.CP.4)\) | ---         | 1 point |
| 51            | Multiple Choice | Create equations that describe numbers or relationships. | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. \((A.CED.1)\) | A          | 1 point |

*The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

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Integrated Math II
Spring 2018 Item Release

Question 1

Question and Scoring Guidelines
Question 1

The graph of a function is shown.

What is the maximum value of the function?
Scoring Guidelines

Exemplar Response

- 3

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).
Integrated Math II
Spring 2018 Item Release

Question 1

Sample Responses
Sample Response: 1 point

The graph of a function is shown.

What is the maximum value of the function?

3
**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct maximum value of the function.

Since the graph (parabola) opens downwards, the vertex (1, 3) is the highest point of the graph and its $y$-coordinate is the maximum value. Therefore, the maximum value of the function is 3.
Sample Response: 1 point

What is the maximum value of the function?

3.0

Notes on Scoring

This response earns full credit (1 point) because it shows a correct equivalent maximum value of the function.
The graph of a function is shown.

What is the maximum value of the function?

1
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect maximum value of the function.

The student may recognize that the graph (parabola) opens downwards and the vertex (1, 3) is the highest point of the graph; however, the student may incorrectly conclude that the x-coordinate of the vertex represents the maximum value of the function.
The graph of a function is shown.

What is the maximum value of the function?

4
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect maximum value of the function.

The student may recognize that the graph (parabola) opens downwards and the vertex is the highest point of the graph but misreads the correct vertex (1, 3) as (1, 4) and uses its y-coordinate, 4, as the maximum value of the function.
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Question 3

Question and Scoring Guidelines
Question 3

Circles R and S are shown. Arc AB has the same length as arc DE.

What is the radius of circle S?

Points Possible: 1

**Content Cluster:** Find arc lengths and areas of sectors of circles.

**Content Standard:** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G.C.5)
Scoring Guidelines

Exemplar Response

- 2

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct radius (1 point).
Sample Response: 1 point

Circles R and S are shown. Arc AB has the same length as arc DE.

What is the radius of circle S?

2.0
Notes on Scoring

This response earns full credit (1 point) because it shows a correct radius of circle S.

In a circle with radius \( r \), the length, \( L \), of the arc intercepted by the central angle \( A \), in degrees, can be calculated by the formula \( L = \frac{A}{180}\pi r \).

In this situation, the two circular sectors with non-equal radii are formed by non-congruent central angles but have equal arc lengths.

By defining the radius of the smaller circle as \( x \) and by substituting 8 for the radius of the larger circle and 30° and 120° for the central angle measures in the formula for the arc length, the situation can be modeled by the equation

\[
\frac{30}{180} \pi \cdot 8 = \frac{120}{180} \pi x
\]

and solved as shown.

\[
\frac{30}{180} \pi \cdot 8 = \frac{120}{180} \pi x
\]

\[
\frac{180}{\pi} \left( \frac{30}{180} \cdot 8 \right) = \left( \frac{120}{180} \pi x \right) \frac{180}{\pi}
\]

\[
30 \cdot 8 = 120 \cdot x
\]

\[
3 \cdot 8 = 12 \cdot x
\]

\[
2 = x
\]

Therefore, the radius of circle S is 2.
Sample Response: 1 point

Notes on Scoring

This response earns full credit (1 point) because it shows an equivalent correct value for the radius of circle $S$. 
Sample Response: 0 points

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for the radius of circle S. The student may overlook the fact that circle R has a radius of 8 and uses a radius of 1 in the calculations instead.
Sample Response: 0 points

This response earns no credit (0 points) because it shows an incorrect value for the radius of circle S. The student may create an incorrect equation \( \frac{360}{30} \pi \cdot 8 = \frac{360}{120} \pi x \) and solves it for \( x \) to get \( x = 32 \).
Question 4

An equation is given.

\[ A = 4 \pi r^2 \]

Solve the equation for \( r \).

A. \( r = \sqrt{\frac{4 \pi}{A}} \)

B. \( r = \sqrt{\frac{A}{4 \pi}} \)

C. \( r = \frac{4 \pi A}{2} \)

D. \( r = \frac{A}{2 \pi} \)

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law \( V = IR \) to highlight resistance \( R \). (A.CED.4)
Scoring Guidelines

Rationale for Option A: The student may incorrectly divide both sides of the equation by $A$, instead of dividing both sides of the equation by $4\pi$, and then takes the square root to solve the equation for $r$.

Rationale for Option B: Key - The student correctly solves the equation for $r$ by dividing both sides of the equation by $4\pi$ to get $r^2 = \frac{A}{4\pi}$ and then by taking the square root of both sides, $\sqrt{\frac{A}{4\pi}} = \sqrt{r^2}$ or $\sqrt{\frac{A}{4\pi}} = r$.

Rationale for Option C: In an attempt to isolate $r^2$, the student may incorrectly multiply the left side of the equation by $4\pi$ and divide the right side of the equation by $4\pi$ to get $4\pi A = r^2$. Then, the student may divide the left side by 2 instead of taking the square root of both sides to get $r = \frac{4\pi A}{2}$.

Rationale for Option D: In the attempt to isolate $r^2$, the student may correctly divide both sides of the equation by $4\pi$ to get $r^2 = \frac{A}{4\pi}$, but then instead of applying the square root to both sides, the student may incorrectly take the square root of the number 4 to get $r = \frac{A}{2\pi}$. 
Sample Response: 1 point

An equation is given.

\[ A = 4\pi r^2 \]

Solve the equation for \( r \).

\[ \text{(A) } r = \sqrt{\frac{4\pi}{A}} \]
\[ \text{(B) } r = \sqrt{\frac{A}{4\pi}} \]
\[ \text{(C) } r = \frac{4\pi A}{2} \]
\[ \text{(D) } r = \frac{A}{2\pi} \]
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Question 5

Question and Scoring Guidelines
Question 5

The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

<table>
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</thead>
<tbody>
<tr>
<td>2010</td>
<td>324</td>
<td>303</td>
<td>340</td>
<td>327</td>
<td>325</td>
<td>338</td>
<td>346</td>
<td>359</td>
<td>350</td>
<td>342</td>
<td>337</td>
<td>326</td>
<td>4017</td>
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<tr>
<td>2011</td>
<td>322</td>
<td>299</td>
<td>330</td>
<td>315</td>
<td>328</td>
<td>335</td>
<td>348</td>
<td>362</td>
<td>346</td>
<td>331</td>
<td>328</td>
<td>322</td>
<td>3966</td>
</tr>
</tbody>
</table>

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

Points Possible: 1

Content Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Content Standard: Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. (S.CP.6)
Scoring Guidelines

Exemplar Response

- \( \frac{299}{3966} \)

Other Correct Responses

- Any equivalent value
- Any decimal between 0.075 and 0.0754

For this item, a full-credit response includes:

- A correct response (1 point).
Sample Response: 1 point

The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

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</tr>
</tbody>
</table>

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

\[
\frac{299}{3966}
\]

Notes on Scoring

This response earns full credit (1 point) because it shows the correct probability for a baby being born in February 2011. To calculate the correct probability, it may be helpful to reword the question as, “What is the probability that the baby was born in February, given that the baby was born in 2011?” Since the total number of babies who were born in 2011 was 3966 and 299 of those babies were born in February, the probability that the baby was born in February 2011 is \( \frac{299}{3966} \).
Sample Response: 1 point

The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

<table>
<thead>
<tr>
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A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

0.075

Notes on Scoring

This response earns full credit (1 point) because it shows the correct equivalent decimal form, 0.075, of the probability for a baby being born in February 2011.
Sample Response: 0 points

The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

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</tr>
</tbody>
</table>

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

\[
\frac{303}{4017}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability for the baby being born in February 2011. The student may view the wrong row of the table and mistakenly calculates the probability of the baby being born in February 2010.
Sample Response: 0 points

The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

<table>
<thead>
<tr>
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</tr>
</tbody>
</table>

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

299000

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability for the baby being born in February 2011. The student may provide the number of babies, 299000, born in February 2011 instead of the probability for the randomly selected baby being born in February 2011.
Integrated Math II
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Question 6

Question and Scoring Guidelines

37 (2018)
Question 6

Jessica writes a proof as shown.

Given: JKLM is a quadrilateral.
- JK ≅ LM
- JK || LM

Prove: ΔKLJ ≅ ΔMJL

Proof:
- JK ≅ LM and JK || LM by given information.
- ∠KJL ≅ ∠MLJ by alternate interior angle theorem.
- JL ≅ JL by the reflexive property.

Which statement is a valid conclusion for Jessica’s proof?

A. ΔKLJ ≅ ΔMJL by SSS.
B. ΔKLJ ≅ ΔMJL by SAS.
C. ΔKLJ ≅ ΔMJL by SSA.
D. ΔKLJ ≅ ΔMJL by AAA.

Points Possible: 1

Content Cluster: Prove theorems involving similarity.

Content Standard: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G.SRT.5)
Scoring Guidelines

Rationale for Option A: The student may notice 3 statements about segments' side lengths in the proof and incorrectly assume that the two triangles are congruent by SSS, not realizing that one of the statements is about two sides being parallel.

Rationale for Option B: Key – The student selects the correct conclusion for the proof and the correct reason. Since two pairs of corresponding sides are congruent and the included angles for these two pairs of sides are congruent because they are alternate interior angles formed by two parallel lines and a transversal, the two triangles are congruent by SAS.

Rationale for Option C: The student may correctly notice two pairs of corresponding congruent sides and a pair of alternate interior angles being congruent but does not realize that the congruence criteria must contain included angles, SAS, for the two pairs of congruent sides.

Rationale for Option D: The student may realize that a pair of alternate interior angles are congruent but incorrectly assume that this criterion is sufficient to prove congruency by AAA, forgetting that AAA can only be used to prove similarity of triangles.
Sample Response: 1 point

Jessica writes a proof as shown.

![Diagram of quadrilateral JKLJ with given segments and angles]

**Given:** JKLJ is a quadrilateral.

- JK \cong LM
- JK \parallel LM

**Prove:** \( \triangle KJL \cong \triangle MJL \)

**Proof:**
- JK \cong LM and JK \parallel LM by given information.
- \( \angle KJL \cong \angle MLJ \) by alternate interior angle theorem.
- JL \cong JL by the reflexive property.

Which statement is a valid conclusion for Jessica’s proof?

A. \( \triangle KJL \cong \triangle MJL \) by SSS.

B. \( \triangle KJL \cong \triangle MJL \) by SAS.

C. \( \triangle KJL \cong \triangle MJL \) by SSA.

D. \( \triangle KJL \cong \triangle MJL \) by AAA.
Question 9

A right triangle ABC is shown.

What is \( \cos A \)?

Points Possible: 1

**Content Cluster:** Define trigonometric ratios and solve problems involving right triangles.

**Content Standard:** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. \((G.SRT.6)\)
Scoring Guidelines

Exemplar Response

- $\frac{8}{17}$

Other Correct Responses

- Any equivalent value
- Any equivalent value between 0.47 and 0.471, inclusive

For this item, a full-credit response includes:

- A correct value (1 point).
Sample Response: 1 point

A right triangle ABC is shown.

What is \( \cos A \)?

0.47

Notes on Scoring

This response earns full credit (1 point) because it shows the correct value of \( \cos A \). For right triangles, the cosine of an acute angle is the ratio of the lengths of the adjacent leg to the hypotenuse. Based on this definition, \( \cos A = \frac{AB}{AC} = \frac{8}{17} \). Any value between 0.47 and 0.471 is also accepted.
Sample Response: 1 point

A right triangle ABC is shown.

What is \( \cos A \)?

0.471

Notes on Scoring

This response earns full credit (1 point) because it shows the correct value of \( \cos A \). For right triangles, the cosine of an acute angle is the ratio of the lengths of the adjacent leg to the hypotenuse. Based on this definition, \( \cos A = \frac{AB}{AC} \), or \( \frac{8}{17} \). Any value between 0.47 and 0.471 is also accepted.
Sample Response: 0 points

A right triangle ABC is shown.

What is \( \cos A \)?

\[ \frac{15}{17} \text{, or approximately 0.88.} \]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for \( \cos A \). The student may confuse the right triangle definition of \( \cos A \) with the right triangle definition of \( \sin A \). In right triangles, the sine of an acute angle is the ratio of the lengths of the opposite leg to the hypotenuse. For this triangle, \( \sin A = \frac{15}{17} \), or approximately 0.88.
Sample Response: 0 points

A right triangle ABC is shown.

What is \( \cos A \)?

2.125

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for \( \cos A \). The student may confuse the ratio used for the right triangle definition of the cosine of an acute angle, and calculates it as the ratio of the lengths of the hypotenuse, 17, to the adjacent leg, 8, or 2.125 instead of the ratio of the lengths of the adjacent leg to the hypotenuse.
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Question 10

Question and Scoring Guidelines
Question 10

The graph of a function $f(x)$ is shown.

Which function has a greater maximum value than $f(x)$?

A. $g(x) = -(x - 1)^2 + 5$
B. $g(x) = -(x - 5)^2 + 1$
C. $g(x) = -(x + 6)^2 - 3$
D. $g(x) = -(x + 3)^2 - 6$

Points Possible: 1

Content Cluster: Analyze functions using different representations.

Content Standard: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (F.IF.9)
Scoring Guidelines

Rationale for Option A: **Key** - The student correctly compares the maximum values of $f(x)$ and $g(x)$. The graph of $f(x)$ has a vertex at (3, 4) and a maximum value of 4. All of the options for $g(x)$ are quadratic functions shown in vertex form, $g(x) = -(x - h)^2 + k$, where $(h, k)$ is the vertex and $k$ is a maximum value. Thus, the function $g(x) = -(x - 1)^2 + 5$ has a vertex at (1, 5) and a maximum value of 5, which is greater than 4.

Rationale for Option B: The student may incorrectly think that the maximum value of the function is represented by the $x$-coordinate of the vertex. The function $f(x)$ has a vertex at (3, 4) and the function $g(x)$ has a vertex at (5, 1). The student may compare 3 and 5 and incorrectly concludes that $g(x)$ has a greater maximum value than $f(x)$.

Rationale for Option C: The student may correctly identify that the maximum value of $f(x)$ is 4 but incorrectly chooses $g(x)$, focusing on the 6 in the formula for $g(x)$, which is larger than 4. The function $g(x)$ has a vertex at (–6, –3) and a maximum value of –3, which is smaller than the maximum value of $f(x)$.

Rationale for Option D: The student may correctly identify that the maximum value of $f(x)$ is 4 but incorrectly chooses $g(x)$, focusing on the 6 in the formula for $g(x)$, which is larger than 4. The function $g(x)$ has a vertex at (–3, –6) and a maximum value of –6, which is smaller than the maximum value of $f(x)$.
The graph of a function $f(x)$ is shown.

Which function has a greater maximum value than $f(x)$?

- $g(x) = -(x - 1)^2 + 5$
- $g(x) = -(x - 5)^2 + 1$
- $g(x) = -(x + 6)^2 - 3$
- $g(x) = -(x + 3)^2 - 6$
Question 11

A system of equations is given.

\[ y = x^2 - 9 \]
\[ y = -2x - 1 \]

What is one solution to the system of equations?

( , )

**Points Possible:** 1

**Content Cluster:** Solve systems of equations.

**Content Standard:** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line \( y = -3x \) and the circle \( x^2 + y^2 = 3 \). (A.REI.7)*
Scoring Guidelines

Exemplar Response

- (-4, 7)

Other Correct Responses

- (2, -5)

For this item, a full-credit response includes:

- A correct ordered pair solution (1 point).
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Question 11

Sample Responses
Sample Response: 1 point

A system of equations is given.

\[ y = x^2 - 9 \]
\[ y = -2x - 1 \]

What is one solution to the system of equations?

\((-4, 7)\)
Notes on Scoring

This response earns full credit (1 point) because it shows a correct solution to a system consisting of a linear equation and a quadratic equation in two variables. There are many methods to solve systems of equations, including substitution. To solve by substitution, first substitute $\pi^2 - 9$ for $y$ in the equation $y = -2x - 1$. The resulting equation becomes $\pi^2 - 9 = -2x - 1$.

Next, apply the Addition Property of Equality by adding $2x + 1$ to both sides of the equation to obtain $\pi^2 + 2x - 8 = 0$. Then, factor the quadratic trinomial on the left side of the equation. When factorization is complete, the equation becomes $\pi^2 + 2\pi - 8 = 0$.

By the Zero-Product Property, for the product $(\pi + 4)(\pi - 2)$ to be equal to 0, one of the factors must be equal to 0, so $\pi + 4 = 0$ or $\pi - 2 = 0$.

Then solve each linear equation for $\pi$, so that $\pi = -4$ or $\pi = 2$. Since the solution to the system of two equations with two variables is the set of ordered pairs consisting of the $\pi$-value and the $y$-value, the final step of the solution process is to substitute the $\pi$-values, one at a time, into one of the original equations to determine the corresponding $y$-values.

For example, when substituting $\pi = -4$ into $y = -2\pi - 1$, the $y$-value is 7, and the solution to the system is $(-4, 7)$.

When substituting $\pi = 2$ into $y = -2\pi - 1$, the $y$-value is $-5$, and the solution is $(2, -5)$. This system has two solutions, but for full credit, the item asks for only one correct ordered pair.
Sample Response: 1 point

A system of equations is given.

\[ y = x^2 - 9 \]
\[ y = -2x - 1 \]

What is one solution to the system of equations?

\[(2, -5)\]

Notes on Scoring

This response earns full credit (1 point) because it shows one out of two correct solutions to the system consisting of a linear equation and a quadratic equation in two variables.
Sample Response: 0 points

A system of equations is given.

\[ y = x^2 - 9 \]
\[ y = -2x - 1 \]

What is one solution to the system of equations?

\((\underline{-5}, \underline{2})\)

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect solution to the system consisting of a linear equation and a quadratic equation in two variables. The system has two solutions, \((2, -5)\) and \((-4, 7)\). This response reverses the order of the coordinates for \(x\) and \(y\) of the first solution and is therefore an incorrect solution.
Sample Response: 0 points

A system of equations is given.

\[ y = x^2 - 9 \]
\[ y = -2x - 1 \]

What is one solution to the system of equations?

\[ (1.633, -6.333) \]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect solution to the system consisting of a linear equation and a quadratic equation in two variables. The student may accidentally change the linear equation \( y = -2x - 1 \) to the quadratic equation \( y = 2x^2 - 1 \) and solves the wrong system of equations.
**Question 14**

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
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</tbody>
</table>

**Points Possible:** 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Content Standard:** Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model. (S.CP.7)
Scoring Guidelines

Exemplar Response

• 0.8

Other Correct Responses

• Any equivalent value

For this item, a full-credit response includes:

• A correct probability (1 point).
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Question 14

Sample Responses
Sample Response: 1 point

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

0.8

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
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<td>0</td>
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</tbody>
</table>
Notes on Scoring

This response earns full credit (1 point) because it shows the correct probability that a student plays soccer or is a girl. One way to calculate this probability is to represent the situation by a two-way frequency table and then use the entries from the table and the Addition Rule for the probability of compound events.

Following from the given information (bold in the table), the total number of students playing either soccer or lacrosse is 50 (bottom right cell), the frequency of “boys playing soccer or lacrosse” is 20 (row 1, column 3) the frequency of “students playing soccer” is 20 (row 3, column 1), and the frequency of “girls playing lacrosse” is 20 (row 2, column 2).

The next step is to calculate the frequency of “Students that are girls” as 50 – 20 = 30 (row 2, column 3) and the frequency of “Students that are girls and play soccer” as 30 – 20 = 10 (row 2, column 1).

<table>
<thead>
<tr>
<th></th>
<th>Soccer</th>
<th>Lacrosse</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Girls</td>
<td>10 (A and B)</td>
<td>20</td>
<td>30 (B)</td>
</tr>
<tr>
<td>Total</td>
<td>20 (A)</td>
<td>30</td>
<td>50</td>
</tr>
</tbody>
</table>

Next, by applying the Addition Rule for the probability of compound events, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), where
- \( P(A) \) - the probability that a student plays soccer \( \frac{20}{50} \)
- \( P(B) \) - the probability that a student is a girl \( \frac{30}{50} \)
- \( P(A \text{ and } B) \) - the probability that a student plays soccer and is a girl \( \frac{10}{50} \)

These values are substituted into the Addition Rule formula to complete the calculation of \( P(A \text{ or } B) \).

\[
P(A \text{ or } B) = \frac{20}{50} + \frac{30}{50} - \frac{10}{50} = \frac{40}{50} = 0.8
\]
Sample Response: 1 point

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

\[
\frac{4}{5}
\]

Notes on Scoring

This response earns full credit (1 point) because it shows a correct probability that a student plays soccer or is a girl in fractional form, \( \frac{4}{5} \), equivalent to the decimal form, 0.8.
Sample Response: 0 points

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

\[
\frac{30}{50}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability that a student plays soccer or is a girl. The student may calculate the probability of randomly choosing a girl, 30, out of a total of 50 students.
Sample Response: 0 points

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

0.4

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability that a student plays soccer or is a girl. The student may calculate the probability of randomly choosing a student who plays soccer, 20, out of a total of 50 students, or \( \frac{20}{50} \), which is 0.4 in decimal form.
Question 20

Events A and B are independent.

\[ P(A \text{ and } B) = 0.25 \]

Enter possible probabilities for events A and B.

\[ P(A) = \]

\[ P(B) = \]

Points Possible: 1

**Content Cluster:** Understand independence and conditional probability and use them to interpret data.

**Content Standard:** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. \((S.CP.2)\)
Scoring Guidelines

Exemplar Response

- \( P(A) = .5 \)
- \( P(B) = .5 \)

Other Correct Responses

- Any values such that \( P(A) \times P(B) = 0.25 \) and both values are greater than 0 and less than or equal to 1

For this item, a full-credit response includes:

- Two correct probabilities (1 point).
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Question 20

Sample Responses
Sample Response: 1 point

Events A and B are independent.

\[ P(A \text{ and } B) = 0.25 \]

Enter possible probabilities for events A and B.

\[
\begin{align*}
P(A) &= 0.4 \\
P(B) &= 0.625
\end{align*}
\]
Notes on Scoring

This response earns full credit (1 point) because it shows two correct possible probabilities for the independent events A and B.

For two independent events A and B, the probability of the intersection of the events can be calculated using the equation \( P(A \text{ and } B) = P(A) \cdot P(B) \). Overall, the probability of an event occurring is greater than or equal to 0 and less than or equal to 1.

In this open-ended situation, since the product of two probabilities is 0.25, each individual probability must be a value that is positive and less than or equal to 1. Therefore, any two values that are positive and less than or equal to 1 whose product is 0.25 could be the probabilities of these two independent events. For example, \( P(A) \) can be 0.4 and \( P(B) \) can also be 0.625 because \( 0.4 \cdot 0.625 = 0.25 \).
Sample Response: 1 point

Events A and B are independent.

\[ P(A \text{ and } B) = 0.25 \]

Enter possible probabilities for events A and B.

\[ P(A) = 0.25 \]
\[ P(B) = 1.0 \]

Notes on Scoring

This response earns full credit (1 point) because it shows two correct possible probabilities for the independent events A and B.

Since the product of the two probabilities is 0.25, each individual probability must be a value that is positive and less than or equal to 1. Therefore, two positive values such as 0.25 and 1, whose product is 0.25 are possible correct probabilities for the independent events A and B.
Sample Response: 0 points

Events A and B are independent.

\[ P(A \text{ and } B) = 0.25 \]

Enter possible probabilities for events A and B.

\[ P(A) = \quad 0.05 \]
\[ P(B) = \quad 5.0 \]

Notes on Scoring

This response earns no credit (0 points) because it shows one impossible probability, \( P(B) = 5 \). Since the probability of an event occurring is always greater than or equal to 0 and less than or equal to 1, the entire response is incorrect, even though \( P(A) \cdot P(B) = 0.25 \).
Sample Response: 0 points

Events A and B are independent.

\[ P(A \text{ and } B) = 0.25 \]

Enter possible probabilities for events A and B.

\[
\begin{align*}
P(A) &= 0.20 \\
P(B) &= 0.05
\end{align*}
\]

Notes on Scoring

This response earns no credit (0 points) because the product of \( P(A) \) and \( P(B) \) is not 0.25. The student may think that when two events are independent, the probability that both events occur is the sum of their individual probabilities instead of the product of their individual probabilities.
Question 21

An inequality is shown.

$(x - 5)(x + 2) < 0$

Select all of the numbers that belong to the solution set of this inequality.

☐ −3
☐ −2
☐ −1
☐ 2
☐ 5

Points Possible: 1

**Content Cluster:** Create equations that describe numbers or relationships.

**Content Standard:** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A.CED.1)
Scoring Guidelines

Rationale for First Option: The student may incorrectly evaluate the inequality \((x - 5)(x + 2) < 0\) for \(x = -3\) as \((-3 - 5)(-3 + 2) = -8 \cdot 1 = -8 < 0\) instead of \(8 > 0\).

Rationale for Second Option: The student may incorrectly substitute \(x = -2\) into the weak inequality \((x - 5)(x + 2) \leq 0\) instead of the strict inequality \((x - 5)(x + 2) < 0\), realizing that when the weak inequality is evaluated for \(x = -2\), the result is a true statement, \(0 = 0\), which makes \(-2\) a solution.

Rationale for Third Option: **Key** - The student correctly evaluates the inequality \((x - 5)(x + 2) < 0\) for \(x = -1\) to get \((-1 - 5)(-1 + 2) = -6 \cdot 1 = -6\) and correctly notes that \(-6 < 0\).

Rationale for Fourth Option: **Key** - The student correctly evaluates the inequality \((x - 5)(x + 2) < 0\) for \(x = 2\) to get \((2 - 5)(2 + 2) = -3 \cdot 4 = -12\) and correctly notes that \(-12 < 0\).

Rationale for Fifth Option: The student may incorrectly substitute \(x = 5\) into the weak inequality \((x - 5)(x + 2) \leq 0\) instead of the strict inequality \((x - 5)(x + 2) < 0\), realizing that when the weak inequality is evaluated for \(x = 5\), the result is a true statement, \(0 = 0\), which makes \(5\) a solution.

Sample Response: 1 point

An inequality is shown.

\((x - 5)(x + 2) < 0\)

Select all of the numbers that belong to the solution set of this inequality.

- [ ] \(-3\)
- [ ] \(-2\)
- [x] \(-1\)
- [x] \(2\)
- [ ] \(5\)
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Question 23

Question and Scoring Guidelines
Question 23

The length of the curve of a satellite dish can be modeled by the function $f(d)$, where $d$ is the horizontal distance, in inches (in.), from the left edge of the satellite dish shown.

What is the domain of the function?

- A) all real numbers from 0 to 18
- B) all real numbers greater than 0
- C) all real numbers from 0 to 5.05
- D) all real numbers from 5.05 to 18

Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context.

Content Standard: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)
Scoring Guidelines

Rationale for Option A: **Key** - The student correctly identifies that the domain for this function describes the horizontal measurement of the dish and is the set of real values bounded by 0 and 18 inches.

Rationale for Option B: The student may correctly identify that the domain for this function must include values that are greater than 0 but does not realize that the horizontal distance from the left side of the dish cannot exceed 18 inches and selects all real numbers greater than zero.

Rationale for Option C: The student may confuse the domain and range of a function that would model the curve. The range is the set of values for the depth of the satellite dish. The domain is the set of values that describes a horizontal distance from the left edge of the satellite dish to a point on the satellite dish.

Rationale for Option D: The student may use the two given numbers in the graphic to represent the domain.

**Sample Response: 1 point**

The length of the curve of a satellite dish can be modeled by the function \( f(d) \), where \( d \) is the horizontal distance, in inches (in.), from the left edge of the satellite dish shown.

What is the domain of the function?

- all real numbers from 0 to 18
- all real numbers greater than 0
- all real numbers from 0 to 5.05
- all real numbers from 5.05 to 18
Question 34

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, $x$.

\[ f(x) = (x - 22)(53 - x) \]

A. What is the minimum price, in dollars, to avoid a loss?

B. What is the maximum price, in dollars, to avoid a loss?

C. What is the price, in dollars, that results in the greatest profit?

A. $\_\_\_\_$

B. $\_\_\_\_$

C. $\_\_\_\_$

Points Possible: 2

**Content Cluster**: Interpret functions that arise in applications in terms of the context.

**Content Standard**: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)
Scoring Guidelines

Exemplar Response

• A. 22
  B. 53
  C. 37.50

Other Correct Responses

• Any equivalent value for each part

For this item, a full-credit response includes:

• The correct minimum and maximum prices (1 point)
  AND
• The correct profit maximizing price (1 point).
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Question 34

Sample Responses
Sample Response: 2 points

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, \( x \).

\[ f(x) = (x - 22)(53 - x) \]

A. What is the minimum price, in dollars, to avoid a loss?

B. What is the maximum price, in dollars, to avoid a loss?

C. What is the price, in dollars, that results in the greatest profit?

A. $22.00

B. $53.00

C. $37.50
This response earns full credit (2 points) because it shows all three correct values that could be determined by graphing a function and then correctly interpreting its key features. The graph of the function \( f(x) = (x - 22)(53 - x) \) is a parabola that opens downwards as shown.

Since the variable \( x \) represents the price, in dollars, of the company's product, and \( y \) represents the profit, in dollars, the part of the parabola that is above the \( x \)-axis represents a positive profit and the part of the parabola that is below the \( x \)-axis represents a loss. The left \( x \)-intercept, or \( x = 22 \), is the minimum price to break even and avoid a loss. The right \( x \)-intercept, or \( x = 53 \), is the maximum price to break even and avoid a loss. The price that results in the greatest profit is the \( x \)-coordinate of the maximum point of the parabola or the \( x \)-coordinate of the vertex that is \( x = 37.50 \).
Sample Response: 2 points

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, \( x \).

\[ f(x) = (x - 22)(53 - x) \]

A. What is the minimum price, in dollars, to avoid a loss?

B. What is the maximum price, in dollars, to avoid a loss?

C. What is the price, in dollars, that results in the greatest profit?

A. \( \$ \, 22 \)

B. \( \$ \, 53 \)

C. \( \$ \, 37.5 \)

Notes on Scoring

This response earns full credit (2 points) because it shows all three correct values.
Sample Response: 1 point

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, \( x \).

\[ f(x) = (x - 22)(53 - x) \]

A. What is the minimum price, in dollars, to avoid a loss?

B. What is the maximum price, in dollars, to avoid a loss?

C. What is the price, in dollars, that results in the greatest profit?

A. \$ 22

B. \$ 53

C. \$ 38

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct minimum and maximum prices only. This student incorrectly rounds the answer for the greatest profit to the nearest dollar amount.
Sample Response: 1 point

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, \( x \).

\[ f(x) = (x - 22)(53 - x) \]

A. What is the minimum price, in dollars, to avoid a loss?

B. What is the maximum price, in dollars, to avoid a loss?

C. What is the price, in dollars, that results in the greatest profit?

\[
\begin{array}{ccc}
A. \$ & 22 & \\
B. \$ & 53 & \\
C. \$ & 31 & \\
\end{array}
\]

Notes on Scoring

This response earns partial credit (1 point) because it shows the correct minimum and maximum prices only. To find the price resulting in the greatest profit, the student may subtract the minimum price from the maximum price instead of finding the average of those prices.
Sample Response: 0 points

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, $x$.

$$f(x) = (x - 22)(53 - x)$$

A. What is the minimum price, in dollars, to avoid a loss?

B. What is the maximum price, in dollars, to avoid a loss?

C. What is the price, in dollars, that results in the greatest profit?

A. $\$ 0$

B. $\$ 0$

C. $\$ 240.25$

Notes on Scoring

This response earns no credit (0 points) because it shows all three incorrect values. The student may provide the $y$-coordinates instead of $x$-coordinates for the three points.
Sample Response: 0 points

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, \( x \).

\[ f(x) = (x - 22)(53 - x) \]

A. What is the minimum price, in dollars, to avoid a loss?

B. What is the maximum price, in dollars, to avoid a loss?

C. What is the price, in dollars, that results in the greatest profit?

\[ \begin{array}{c}
A. \$ \quad 23 \\
B. \$ \quad 52 \\
C. \$ \quad 36.5
\end{array} \]

Notes on Scoring

This response earns no credit (0 points) because it shows all three incorrect values. The student may misinterpret the wording of “to avoid loss” to mean that the price needs to be greater than 0 and provide integer values, 23 and 52, consecutive to the two correct values 22 and 53. Then the student may find the mean of the incorrect values in part A and B to determine the price that results in the greatest profit.
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Question 35

Question and Scoring Guidelines
Question 35

An expression is shown.

\[(3x^2 + 2x - 5) - (5x^2 - 4x + 1)\]

Which expression is equivalent to the given expression?

A \[ -2x^2 + 6x - 6 \]

B \[ -2x^2 - 2x - 4 \]

C \[ 8x^2 + 6x - 6 \]

D \[ 8x^2 - 2x - 4 \]

Points Possible: 1

**Content Cluster:** Perform arithmetic operations on polynomials.

**Content Standard:** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1)
Scoring Guidelines

Rationale for Option A: Key - The student correctly subtracts the polynomials as 
\((3\pi^2 + 2\pi - 5) - (5\pi^2 - 4\pi + 1) = 3\pi^2 + 2\pi - 5 - 5\pi^2 + 4\pi - 1 = -2\pi^2 + 6\pi - 6\)

Rationale for Option B: The student may only distribute the negative to the first term of the second polynomial as
\((3\pi^2 + 2\pi - 5) - (5\pi^2 - 4\pi + 1) = 3\pi^2 + 2\pi - 5 - 5\pi^2 - 4\pi + 1 = -2\pi^2 - 2\pi - 4\)

Rationale for Option C: The student may incorrectly distribute the negative to the \(x\)-term and the constant term but not to the \(x^2\) term of the second polynomial as
\((3\pi^2 + 2\pi - 5) - (5\pi^2 - 4\pi + 1) = 3\pi^2 + 2\pi - 5 + 5\pi^2 + 4\pi - 1 = 8\pi^2 + 6\pi - 6\)

Rationale for Option D: The student may add the polynomials instead of subtracting them as
\((3\pi^2 + 2\pi - 5) + (5\pi^2 - 4\pi + 1) = 3\pi^2 + 2\pi - 5 + 5\pi^2 - 4\pi + 1 = 8\pi^2 - 2\pi - 4\)

Sample Response: 1 point

An expression is shown.

\((3\pi^2 + 2\pi - 5) - (5\pi^2 - 4\pi + 1)\)

Which expression is equivalent to the given expression?

- \(\text{a} - 2\pi^2 + 6\pi - 6\)
- \(\text{b} - 2\pi^2 - 2\pi - 4\)
- \(\text{c} 8\pi^2 + 6\pi - 6\)
- \(\text{d} 8\pi^2 - 2\pi - 4\)
Question 36

Right triangle FHG is shown.

The sine of $\angle F$ is 0.53.

What is the cosine of $\angle H$? Round your answer to the nearest hundredth as needed.

Points Possible: 1

Content Cluster: Define trigonometric ratios and solve problems involving right triangles.

Content Standard: Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)
**Scoring Guidelines**

**Exemplar Response**
- 0.53

**Other Correct Responses**
- Any equivalent value

For this item, a full-credit response includes:
- A correct value (1 point).
Sample Response: 1 point

Right triangle FHG is shown.

The sine of \( \angle F \) is 0.53.

What is the cosine of \( \angle H \)? Round your answer to the nearest hundredth as needed.

0.53

Notes on Scoring

This response earns full credit (1 point) because it shows the correct value for \( \cos(H) \). In right triangles, the sine of an acute angle equals the cosine of the angle’s complement. Since angles \( F \) and \( H \) are complementary angles (their sum is 90°), the cosine of \( H \) equals the sine of \( F \), or 0.53.
Sample Response: 1 point

Right triangle FHG is shown.

The sine of \( \angle F \) is 0.53.

What is the cosine of \( \angle H \)? Round your answer to the nearest hundredth as needed.

\[
\frac{53}{100}
\]

Notes on Scoring

This response earns full credit (1 point) because it shows a correct equivalent value for \( \cos(H) \).
Sample Response: 0 points

Right triangle FHG is shown.

The sine of ∠F is 0.53.

What is the cosine of ∠H? Round your answer to the nearest hundredth as needed.

\[
\frac{1}{0.53}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for \(\cos(H)\). The student may incorrectly think that the \(\cos(H)\) is the reciprocal of \(\sin(F)\) because the angles are complementary.
Sample Response: 0 points

Right triangle FHG is shown.

The sine of $\angle F$ is 0.53.

What is the cosine of $\angle H$? Round your answer to the nearest hundredth as needed.

0.47

Notes on Scoring

This response earns no credit (0 points) because it shows the incorrect value for $\cos(H)$. The student may incorrectly think that the $\cos(H) = 1 - \sin(F)$ because the angles are complementary.
Question 39

A cone and a sphere have the same volume. The height of the cone is 96 units. What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: [ ] units
Radius of Sphere: [ ] units

Points Possible: 1

Content Cluster: Explain volume formulas and use them to solve problems.

Content Standard: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G.GMD.3)
**Scoring Guidelines**

**Exemplar Response**

- Radius of cone: 3 units
- Radius of sphere: 6 units

**Other Correct Responses**

- Any values for which $|24(\text{radius of cone})^2 - (\text{radius of sphere})^3| \leq 1$

For this item, a full-credit response includes:

- Two correct values (1 point).
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Question 39

Sample Responses
Sample Response: 1 point

A cone and a sphere have the same volume. The height of the cone is 96 units. What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: 3 units
Radius of Sphere: 6 units
Notes on Scoring

This response earns full credit (1 point) because it shows correct values for the radius of cone \( R \) and the radius of sphere \( r \). In this situation, the student can calculate these values by creating an equation that shows the equality of these two volumes as shown below.

Volume of cone = Volume of sphere
\[
\frac{1}{3} \pi R^2 h = \frac{4}{3} \pi r^3
\]

Using the fact that the height, \( h \), of the cone is 96 units, the equation becomes \( \frac{1}{3} \pi R^2 96 = \frac{4}{3} \pi r^3 \).

After multiplying both sides by \( \frac{3}{4} \) and then dividing both sides by \( \pi \), the equation becomes \( 24 R^2 = r^3 \).

To find solutions to this equation, the student can pick any positive value for the radius of one shape, substitute it into the equation, and calculate the radius of another shape by solving the equation for the unknown. For example, if \( R = 3 \), then \( 24 \cdot 3^2 = r^3 \), \( 216 = r^3 \), and \( r = 6 \); or if \( R = 5 \), then \( 24 \cdot 5^2 = r^3 \), and \( r \approx 8.43 \).

The acceptable values are those that satisfy the condition \( |24 R^2 - r^3| \leq 1 \). Values for both radii can be rounded to the nearest hundredths of units, if necessary.
Sample Response: 1 point

A cone and a sphere have the same volume. The height of the cone is 96 units. What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: 3.0 units

Radius of Sphere: 6.0 units

Notes on Scoring

This response earns full credit (1 point) because it shows correct possible values for the radius of the cone ($R$) and the radius of the sphere ($r$).
Sample Response: 0 points

A cone and a sphere have the same volume. The height of the cone is 96 units.
What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: 3 units
Radius of Sphere: 3 units

Notes on Scoring

This response earns no credit (0 points) because the two values shown do not satisfy the inequality \(|24R^2 - r^3| \leq 1\). The student may think that \(R\) and \(r\) are the same value.
Sample Response: 0 points

A cone and a sphere have the same volume. The height of the cone is 96 units. What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: 48 units
Radius of Sphere: 48 units

Notes on Scoring

This response earns no credit (0 points) because the two values shown do not satisfy the inequality $|24r^2 - r^3| \le 1$. The student may confuse the height of the cone with the diameter of the cone and divides 96 by 2 to get the radius of 48. He or she may also incorrectly think that the radius of the cone and the radius of the sphere are equal and enters 48 for the radius of the sphere as well.
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Question 44

Question and Scoring Guidelines
Mark is proving the Pythagorean Theorem. He draws right triangle JKL with altitude JM. First he proves \( \triangle JKL \sim \triangle MKJ \) and \( \triangle JKL \sim \triangle MJL \) using the Angle-Angle criterion. The rest of his proof is shown with some steps missing.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle JKL \sim \triangle MKJ ) and ( \triangle JKL \sim \triangle MJL )</td>
<td>1. Angle-Angle criterion</td>
</tr>
<tr>
<td>2. ( \frac{JK}{LK} = \frac{MK}{JK} ) and ( \frac{LJ}{LK} = \frac{ML}{LJ} )</td>
<td>2. Corresponding sides of similar triangles are proportional</td>
</tr>
<tr>
<td>3. ( (JK)^2 = LK \cdot MK ) and ( (LJ)^2 = LK \cdot ML )</td>
<td>3. Multiplication property of equality</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( MK + ML = LK )</td>
<td>6. Segment addition postulate</td>
</tr>
<tr>
<td>7. ( (JK)^2 + (LJ)^2 = (LK)^2 )</td>
<td>7. Substitution</td>
</tr>
</tbody>
</table>

Which two steps are missing from the proof?

A. 4. \( (JK)^2 + (LJ)^2 = LK \cdot MK + LK \cdot ML \)  
   5. \( (JK)^2 + (LJ)^2 = LK(MK + ML) \)

B. 4. \( (JK)^2 - (LJ)^2 = LK \cdot MK - LK \cdot ML \)  
   5. \( (JK)^2 - (LJ)^2 = LK(MK - ML) \)

Points Possible: 1

Content Cluster: Prove theorems involving similarity.

Content Standard: Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely: the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)
Scoring Guidelines

Rationale for Option A: **Key** - The student correctly realizes that adding the squares of the two legs of triangle JKL is necessary to complete the proof. Applying the Addition Property of Equality to the left and right sides of the two equations creates the correct equation in step 4. Factoring LK out on the right side (using the Distributive Property) leads to the expression LK(MK + ML), which results in LK • LK after using the Segment Addition Postulate, and concludes the proof of Pythagorean Theorem.

Rationale for Option B: The student correctly identifies that the Addition Property of Equality should be used to add the left and right sides of the 2 equations to create the correct equation for step 4 but may miss that factoring LK out in step 5 would result in LK • (LK + LK) = LK^2, which is not equivalent to the expression in step 6.

Rationale for Option C: The student may think that step 5 involves multiplication instead of addition and then incorrectly factors the expression on the right side to obtain the expression in step 6.

Rationale for Option D: The student may think that step 4 involves multiplication instead of addition and that only LK lengths would be on the right side in step 5, and that LK(LK • LK) = LK^2, which is not equivalent to the expression in step 6.
Mark is proving the Pythagorean Theorem. He draws right triangle JKL with altitude JM. First, he proves ΔJKL ~ ΔMKJ and ΔJKL ~ ΔMJL using the Angle-Angle criterion. The rest of his proof is shown with some steps missing.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ΔJKL ~ ΔMKJ and ΔJKL ~ ΔMJL</td>
<td>1. Angle-Angle criterion</td>
</tr>
<tr>
<td>2. ( \frac{JK}{LM} = \frac{MK}{MJ} ) and ( \frac{JK}{LM} = \frac{ML}{LJ} )</td>
<td>2. Corresponding sides of similar triangles are proportional</td>
</tr>
<tr>
<td>3. ( (JK)^2 = LK \cdot MK ) and ( (LJ)^2 = LK \cdot ML )</td>
<td>3. Multiplication property of equality</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
</tr>
<tr>
<td>6. MK + ML = LK</td>
<td>6. Segment addition postulate</td>
</tr>
<tr>
<td>7. ( (JK)^2 + (LJ)^2 = (LK)^2 )</td>
<td>7. Substitution</td>
</tr>
</tbody>
</table>

Which two steps are missing from the proof?

- 4. \( (JK)^2 + (LJ)^2 = LK \cdot MK + LK \cdot ML \)
  - 4. Addition property of equality
  - 4. Multiplication property of equality

- 5. \( (JK)^2 + (LJ)^2 = LK(MK + ML) \)
  - 5. Distributive property
  - 5. Distributive property
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Question 45

Question and Scoring Guidelines
**Question 45**

An expression is given.

\[(\sqrt[16]{16})^4\]

Which set of steps correctly evaluates this expression?

<table>
<thead>
<tr>
<th></th>
<th>Given</th>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>((\sqrt[16]{16})^4)</td>
<td>((16^{\frac{1}{4}})^4)</td>
<td>((\frac{16}{4})^4)</td>
<td>4^4</td>
<td>256</td>
</tr>
<tr>
<td>B</td>
<td>((\sqrt[16]{16})^4)</td>
<td>((16^{\frac{1}{4}})^4)</td>
<td>16^4</td>
<td>16^1</td>
<td>16</td>
</tr>
<tr>
<td>C</td>
<td>((\sqrt[16]{16})^4)</td>
<td>((16^{\frac{3}{4}})^4)</td>
<td>16^\frac{3}{4}</td>
<td>4 \cdot 4</td>
<td>16</td>
</tr>
<tr>
<td>D</td>
<td>((\sqrt[16]{16})^4)</td>
<td>((16^{\frac{3}{4}})^4)</td>
<td>16^\frac{3}{4} \cdot 4</td>
<td>4 \cdot 4</td>
<td>16</td>
</tr>
</tbody>
</table>

**Points Possible:** 1

**Content Cluster:** Extend the properties of exponents to rational exponents.

**Content Standard:** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define \(5^{\frac{1}{3}}\) to be the cube root of 5 because we want \((5^{\frac{1}{3}})^3 = 5^{\frac{1}{3} \cdot 3}\) to hold, so \((5^{\frac{1}{3}})^3\) must equal 5. (N.RN.1)"
Scoring Guidelines

Rationale for Option A: The student may correctly identify that the 4th root of 16 is equivalent to $16^{\frac{1}{4}}$ but incorrectly thinks that $16^{\frac{1}{4}}$ is equivalent to $\frac{16}{4}$ or 4. Then, the student completes calculations as $4^4 = 256$.

Rationale for Option B: Key – The student correctly identifies that the 4th root of 16 is equivalent to $16^{\frac{1}{4}}$, uses the Power to a Power Property of exponents to write $\left(16^{\frac{1}{4}}\right)^4$ as $16^1$, simplifies the fractional exponent, and then identifies that $16^1 = 16$.

Rationale for Option C: The student may mistake the 4th root symbol for a square root, and thinks that the 4th root of 16 is equivalent to $16^{\frac{1}{2}}$, then uses the Power to a Power Property of exponents to write $\left(16^{\frac{1}{2}}\right)^4$ as $16^{\frac{4}{2}}$, simplifies the fractional exponent, and completes calculations as $16^2 = 256$.

Rationale for Option D: The student may realize that the value of the expression is 16, and so chooses an option that has 16 as the final step, but misses that this set of steps incorrectly assumes that the 4th root of 16 is equivalent to $16^{\frac{1}{2}}$ and that a number to the 4th power is equal to that number multiplied by 4.
An expression is given.

\((\sqrt[4]{16})^4\)

Which set of steps correctly evaluates this expression?

<table>
<thead>
<tr>
<th></th>
<th>Given</th>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>((\sqrt[4]{16})^4)</td>
<td>((16^{\frac{1}{4}})^4)</td>
<td>((\frac{16}{4})^4)</td>
<td>(4^4)</td>
<td>(256)</td>
</tr>
<tr>
<td>B</td>
<td>((\sqrt[4]{16})^4)</td>
<td>((16^{\frac{1}{4}})^4)</td>
<td>(16^{\frac{4}{4}})</td>
<td>(16^2)</td>
<td>(256)</td>
</tr>
<tr>
<td>C</td>
<td>((\sqrt[4]{16})^4)</td>
<td>((16^{\frac{1}{4}})^4)</td>
<td>(16^{\frac{4}{4}})</td>
<td>(16^1)</td>
<td>(16)</td>
</tr>
<tr>
<td>D</td>
<td>((\sqrt[4]{16})^4)</td>
<td>((16^{\frac{1}{4}})^4)</td>
<td>(16^{\frac{4}{4}})</td>
<td>(4\cdot4)</td>
<td>(16)</td>
</tr>
</tbody>
</table>
Question 49

Line segment $AB$ is shown on the coordinate grid. Line segment $AB$ is dilated by a scale factor of 2, with the center of dilation at the origin, resulting in line segment $A'B'$.

Use the Connect Line tool to draw line segment $A'B'$.

**Points Possible:** 1

**Content Cluster:** Understand similarity in terms of similarity transformations.

**Content Standard:** Verify experimentally the properties of dilations given by a center and a scale factor: $(G.SRT.1)$

$a.$ A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
Scoring Guidelines

Exemplar Response

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- The correct line segment (1 point).
Sample Response: 1 point

Line segment $AB$ is shown on the coordinate grid. Line segment $AB$ is dilated by a scale factor of 2, with the center of dilation at the origin, resulting in line segment $A'B'$.

Use the Connect Line tool to draw line segment $A'B'$.

Notes on Scoring

This response earns full credit (1 point) because it shows a line segment, $A'B'$, which is the correct image of the line segment $AB$ on the coordinate grid.

When a line segment $AB$, with endpoints at $A$ (–3, 0) and $B(3, 2)$ undertakes a dilation with the center of dilation at the origin, $O$, by a scale factor 2, the image, $A'$, of the point $A$ remains on the ray $OA$ but twice as far from the origin as point $A$, or at (–6, 0).

The image, $B'$, of the point $B$ remains on the ray $OB$ but twice as far from the origin as point $B$, or at (6, 4). By using the Connect Line tool to connect points $A'$ and $B'$, the student obtains the resulting line segment $A'B'$. 
Sample Response: 1 point

Line segment AB is shown on the coordinate grid. Line segment AB is dilated by a scale factor of 2, with the center of dilation at the origin, resulting in line segment A'B'.

Use the Connect Line tool to draw line segment A'B'.

Notes on Scoring

This response earns full credit (1 point) because it shows a line segment, A'B', which is the correct image of the line segment AB on the coordinate grid.

In addition to the correct line segment A'B', the student leaves some miscellaneous points on the coordinate grid. The presence of these points does not affect the student's score.
Sample Response: 0 points

Line segment AB is shown on the coordinate grid. Line segment AB is dilated by a scale factor of 2, with the center of dilation at the origin, resulting in line segment A'B'.

Use the Connect Line tool to draw line segment A'B'.

Notes on Scoring

This response earns no credit (0 points) because it shows a line segment, A'B', which is an incorrect image of the line segment AB on the coordinate grid.

The student may use point A instead of the origin, point O, as the center of dilation to plot the image of a point B.

When the original line segment AB undertakes a dilation with a center that is not on line segment AB and a scale factor 2, the segment and its image are distinct parallel line segments with equal slopes.
Sample Response: 0 points

Line segment AB is shown on the coordinate grid. Line segment AB is dilated by a scale factor of 2, with the center of dilation at the origin, resulting in line segment A'B'.

Use the Connect Line tool to draw line segment A'B'.

Notes on Scoring

This response earns no credit (0 points) because it shows a line segment, A'B', which is an incorrect image of the line segment AB on the coordinate grid.

The student may use point A instead of point O as the center of dilation and may think that dilating a line segment by a scale factor of 2 will result in multiplying the slope of AB by 2. However, when the original line segment undertakes the dilation with the center at O, the slopes of the image and the original line segment remain equal.
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Question 50

Question and Scoring Guidelines
**Question 50**

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

<table>
<thead>
<tr>
<th></th>
<th>Like Popcorn</th>
<th>Don’t Like Popcorn</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like Candy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Don’t Like Candy</td>
<td></td>
<td></td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>70</td>
<td></td>
<td>120</td>
</tr>
</tbody>
</table>

**Points Possible: 1**

**Content Cluster:** Understand independence and conditional probability and use them to interpret data.

**Content Standard:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)
Scoring Guidelines

Exemplar Response

<table>
<thead>
<tr>
<th></th>
<th>Like Popcorn</th>
<th>Don’t Like Popcorn</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like Candy</td>
<td>42</td>
<td>16</td>
<td>58</td>
</tr>
<tr>
<td>Don’t Like Candy</td>
<td>28</td>
<td>34</td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>70</td>
<td>50</td>
<td>120</td>
</tr>
</tbody>
</table>

Other Correct Responses

- Any equivalent decimal values

For this item, a full-credit response includes:

- A correct table (1 point).
Sample Response: 1 point

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

<table>
<thead>
<tr>
<th></th>
<th>Like Popcorn</th>
<th>Don’t Like Popcorn</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like Candy</td>
<td>42</td>
<td>16</td>
<td>58</td>
</tr>
<tr>
<td>Don’t Like Candy</td>
<td>28</td>
<td>34</td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>70</td>
<td>50</td>
<td>120</td>
</tr>
</tbody>
</table>

Notes on Scoring

This response earns full credit (1 point) because it shows a completely correct two-way frequency table. Using frequencies given in the table, the first step is to determine the blank subtotals as 120 - 70 = 50 to complete the bottom row and 120 - 62 = 58 to complete the last column.

Next, the value in the top left cell of the table can be determined by using the fact that 60% of those who like to eat popcorn also like to eat candy, or 70 • 0.6 = 42.

Lastly, using all known frequencies, the remaining parts of the table can be completed as:

70 - 42 = 28 (second row, first column)
58 - 42 = 16 (first row, second column)
62 - 28 = 34 (second row, second column)
Sample Response: 1 point

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

• Do you like to eat popcorn at the movie theater?
• Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

<table>
<thead>
<tr>
<th></th>
<th>Like Popcorn</th>
<th>Don’t Like Popcorn</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like Candy</td>
<td>42.0</td>
<td>16.0</td>
<td>58.0</td>
</tr>
<tr>
<td>Don’t Like Candy</td>
<td>28.0</td>
<td>34.0</td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>70</td>
<td>50.0</td>
<td>120</td>
</tr>
</tbody>
</table>

Notes on Scoring

This response earns full credit (1 point) because it shows a completely correct two-way frequency table with equivalent entries.
Sample Response: 0 points

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

<table>
<thead>
<tr>
<th></th>
<th>Like Popcorn</th>
<th>Don’t Like Popcorn</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like Candy</td>
<td>60</td>
<td>-2</td>
<td>58</td>
</tr>
<tr>
<td>Don’t Like Candy</td>
<td>10</td>
<td>52</td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>70</td>
<td>50</td>
<td>120</td>
</tr>
</tbody>
</table>

Notes on Scoring

This response earns no credit (0 points) because it shows four incorrect entries in the two-way frequency table. First, the student may correctly determine the blank subtotals as $120 - 70 = 50$ to complete the bottom row and $120 - 62 = 58$ to complete the last column.

Next, the student may incorrectly enter 60 in the top left cell of the table instead of the value representing 60% of those who liked popcorn also liked candy, or $70 \times 0.6 = 42$. Based on this error, the student completes the rest of the table.
Sample Response: 0 points

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

<table>
<thead>
<tr>
<th></th>
<th>Like Popcorn</th>
<th>Don’t Like Popcorn</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like Candy</td>
<td>34.8</td>
<td>23.2</td>
<td>58</td>
</tr>
<tr>
<td>Don’t Like Candy</td>
<td>35.2</td>
<td>26.8</td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>70</td>
<td>50</td>
<td>120</td>
</tr>
</tbody>
</table>

Notes on Scoring

This response earns no credit (0 points) because it shows four incorrect entries in the two-way frequency table. First, the student may correctly determine the blank subtotals as $120 - 70 = 50$ to complete the bottom row and $120 - 62 = 58$ to complete the last column.

Next, the student may mistakenly calculate 60% of those who like candy also liked popcorn, or $58 \times 0.6 = 34.8$, for the top left cell of the table instead of the value representing 60% of those who liked popcorn also liked candy, or $70 \times 0.6 = 42$. Based on this error, the student completes the rest of the table.
Integrated Math II
Spring 2018 Item Release

Question 51

Question and Scoring Guidelines
Question 51

A table with population data, rounded to the nearest thousand, for Ohio in 2010 and 2014 is shown.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>11,536,000</td>
</tr>
<tr>
<td>2014</td>
<td>11,594,000</td>
</tr>
</tbody>
</table>

An Ohio state official claims that the population, $P$, will grow exponentially with respect to time, $t$, in years after 2010.

Based on the claim of the state official and the data in the table, which equation provides the best estimate of the population of Ohio in the year 2020?

@ $P = 11,536,000(1.0013)^t$
@ $P = 11,536,000(1.0203)^t$
@ $P = 11,536,000(1.0013)^6$
@ $P = 11,536,000(1.0203)^6$

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A.CED.1)
**Scoring Guidelines**

**Rationale for Option A: Key** - The student correctly selects an equation that models the data in the table. Since \( t \) is the number of years since 2010, the year 2020 is represented by \( t = 10 \), and the initial population is 11,536,000. Since the population grows exponentially, the student substitutes 11,536,000 for \( N \), 11,594,000 for \( A \) and 4 for \( t \) into the general equation for exponential function, \( A = N(1 + r)^t \), where \( N \) is the initial population, \((1 + r)\) is the growth factor and \( r \) is the growth rate, to obtain the equation 11,594,000 = 11,536,000\((1 + r)^4\). To find the growth factor \((1 + r)\), the student divides both sides of the equation by 11,536,000 to obtain 1.00503 = \((1 + r)^4\). Then, the student takes the fourth root of 1.00503 to determine the base, 1.0013, for the equation modeling the data in the table. After that, the student substitutes these values into the general exponential equation.

**Rationale for Option B:** The student may correctly observe that if \( t \) is the number of years since 2010, \( t = 10 \) represents the year of 2020, and the initial population \( N \) is 11,536,000 but incorrectly calculates the growth factor. The student may divide both sides of the equation 11,594,000 = 11,536,000\((1 + r)^4\) by 11,536,000 to obtain 1.00503 = \((1 + r)^4\). Then, instead of taking the fourth root to find the growth factor, the student mistakenly raises the number to the power of 4 to obtain 1.0203. Next, the student substitutes these values into the general exponential equation.

**Rationale for Option C:** The student may correctly observe that \( t \) represents the number of years after 2010 and that the population in 2010 is the initial population. Since the population grows exponentially, the student correctly substitutes 11,536,000 for \( N \) and 11,594,000 for \( A \) but made an error by subtracting 2014 and 2010 instead of 2020 and 2010 to obtain the equation 11,594,000 = 11,536,000\((1 + r)^4\). To find the growth factor \((1 + r)\), the student divides both sides of the equation by 11,536,000 to get 1.00503 = \((1 + r)^4\). Then, the student takes the fourth root of 1.00503 to determine the approximate base, 1.0013, for the equation modeling the data in the table. When calculating the exponent, the student may mistakenly subtract 2014 from 2020 to get \( t = 6 \), instead of subtracting 2010 from 2020, to get \( t = 10 \). The student then substitutes these values into the general exponential equation.

**Rationale for Option D:** The student may correctly observe that \( t \) represents the number of years after 2010 and that the population in 2010 is the initial population, but incorrectly calculates the value of \( t \) and the growth factor. When calculating the time \( t \), the student may mistakenly subtract 2014 from 2020 to get \( t = 6 \), instead of subtracting 2010 from 2020, or \( t = 10 \). When calculating the growth factor, the student divides both sides of the equation...
11,594,000 = 11,536,000(1 + r)^4 \text{ by } 11,536,000 \text{ to obtain } 1.00503 = (1 + r)^4. \text{ Then, the student mistakenly raises } 1.00503 \text{ to the power of } 4 \text{ (since } 2014 - 2010 = 4) \text{ to obtain } 1.0203. \text{ Next, the student substitutes these values into the general exponential equation.}

**Sample Response: 1 point**

A table with population data, rounded to the nearest thousand, for Ohio in 2010 and 2014 is shown.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>11,536,000</td>
</tr>
<tr>
<td>2014</td>
<td>11,594,000</td>
</tr>
</tbody>
</table>

An Ohio state official claims that the population, \( P \), will grow exponentially with respect to time, \( t \), in years after 2010.

Based on the claim of the state official and the data in the table, which equation provides the best estimate of the population of Ohio in the year 2020?

- \( P = 11,536,000(1.0015)^5 \)
- \( P = 11,536,000(1.0203)^5 \)
- \( P = 11,536,000(1.0013)^6 \)
- \( P = 11,536,000(1.0203)^6 \)