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<th>Answer Key</th>
<th>Points</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Equation Item</td>
<td>Understand the relationships between lengths, area, and volumes.</td>
<td>When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by $k$, $k^2$, and $k^3$, respectively. (G.GMD.6)</td>
<td>Level 2</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>7</td>
<td>Multiple Choice Item</td>
<td>Explain volume formulas and use them to solve problems.</td>
<td>Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G.GMD.1)</td>
<td>Level 3</td>
<td>A</td>
<td>1 point</td>
</tr>
<tr>
<td>9</td>
<td>Matching Item</td>
<td>Interpret the structure of expressions.</td>
<td>Interpret expressions that represent a quantity in terms of its context. (A.SSE.1) b. Interpret complicated expressions by viewing one or more of their parts as a single entity.</td>
<td>Level 2</td>
<td>---</td>
<td>1 point</td>
</tr>
</tbody>
</table>

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<tbody>
<tr>
<td>12</td>
<td>Multi Select Item</td>
<td>Interpret functions that arise in applications in terms of the context.</td>
<td>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. b. Focus on linear, quadratic, and exponential functions. (A1, M2) (F.IF.4)</td>
<td>Level 2</td>
<td>A, F</td>
<td>1 point</td>
</tr>
<tr>
<td>13</td>
<td>Equation Item</td>
<td>Apply geometric concepts in modeling situations.</td>
<td>Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (G.MG.3)</td>
<td>Level 2</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>16</td>
<td>Multiple Choice Item</td>
<td>Construct and compare linear, quadratic, and exponential models and solve problems.</td>
<td>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (A1, M2) (F.LE.3)</td>
<td>Level 2</td>
<td>D</td>
<td>1 point</td>
</tr>
<tr>
<td>20</td>
<td>Multiple Choice Item</td>
<td>Find arc lengths and areas of sectors of circles. (G.C.5) a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.</td>
<td>Find arc lengths and areas of sectors of circles. (G.C.5) a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.</td>
<td>Level 2</td>
<td>A</td>
<td>1 point</td>
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<tr>
<td>22</td>
<td>Multiple Choice Item</td>
<td>Perform arithmetic operations on polynomials.</td>
<td>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) (A.APR.1)</td>
<td>Level 1</td>
<td>C</td>
<td>1 point</td>
</tr>
<tr>
<td>23</td>
<td>Multiple Choice Item</td>
<td>Understand independence and conditional probability, and use them to interpret data.</td>
<td>Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)</td>
<td>Level 2</td>
<td>B</td>
<td>1 point</td>
</tr>
<tr>
<td>24</td>
<td>Equation Item</td>
<td>Prove and apply theorems both formally and informally involving similarity using a variety of methods.</td>
<td>Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. (G.SRT.5)</td>
<td>Level 2</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>26</td>
<td>Multiple Choice Item</td>
<td>Visualize relationships between two-dimensional and three-dimensional objects.</td>
<td>Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G.GMD.4)</td>
<td>Level 1</td>
<td>A</td>
<td>1 point</td>
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<tr>
<td>28</td>
<td>Multiple Choice Item</td>
<td>Build new functions from existing functions.</td>
<td>Identify the effect on the graph of replacing (f(x)) by (f(x) + k), (k f(x)), (f(kx)), and (f(x + k)) for specific values of (k) (both positive and negative); find the value of (k) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. a. Focus on transformations of graphs of quadratic functions, except for (f(kx)); (A1, M2) (F.BF.3)</td>
<td>Level 1</td>
<td>C</td>
<td>1 point</td>
</tr>
<tr>
<td>31</td>
<td>Equation Item</td>
<td>Understand independence and conditional probability, and use them to interpret data.</td>
<td>Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)</td>
<td>Level 2</td>
<td>---</td>
<td>1 point</td>
</tr>
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<tr>
<td>35</td>
<td>Equation Item</td>
<td>Create equations that describe numbers or relationships.</td>
<td>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. c. Focus on formulas in which the variable of interest is linear or square. For example, rearrange the formula for the area of a circle ( A = (\pi)r^2 ) to highlight radius ( r ). (M2) (A.CED.4)</td>
<td>Level 2</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>37</td>
<td>Inline Choice Item</td>
<td>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</td>
<td>Apply the Addition Rule, ( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) ), and interpret the answer in terms of the model. (S.CP.7)</td>
<td>Level 2</td>
<td>---</td>
<td>1 point</td>
</tr>
<tr>
<td>38</td>
<td>Equation Item</td>
<td>Solve equations and inequalities in one variable.</td>
<td>Solve quadratic equations in one variable. (A.REI.4) b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for ( x^2 = 49 ); taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.</td>
<td>Level 3</td>
<td>---</td>
<td>2 points</td>
</tr>
<tr>
<td>39</td>
<td>Multiple Choice Item</td>
<td>Understand similarity in terms of similarity transformations.</td>
<td>Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (G.SRT.3)</td>
<td>Level 1</td>
<td>C</td>
<td>1 point</td>
</tr>
</tbody>
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</table>
| 42            | Equation Item | Write expressions in equivalent forms to solve problems. | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *(A.SSE.3)*  
a. Factor a quadratic expression to reveal the zeros of the function it defines. | Level 3 | --- | 1 point |
| 44            | Equation Item | Analyze functions using different representations. | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *(F.IF.8)*  
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \( y = (1.02)^t \), and \( y = (0.97)^t \) and classify them as representing exponential growth or decay. i. Focus on exponential functions evaluated at integer inputs. *(A1, M2)* | Level 2 | --- | 1 point |
| 45            | Multiple Choice Item | Use the rules of probability to compute probabilities of compound events in a uniform probability model. | Find the conditional probability of \( A \) given \( B \) as the fraction of \( B \)'s outcomes that also belong to \( A \), and interpret the answer in terms of the model. *(S.CP.6)* | Level 2 | D | 1 point |
| 47            | Multiple Choice Item | Define trigonometric ratios and solve problems involving right triangles. | Explain and use the relationship between the sine and cosine of complementary angles. *(G.SRT.7)* | Level 1 | B | 1 point |

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Depth of Knowledge (DOK)

DOK refers to the complexity of thinking required to complete a task in a given item. Items with a DOK 1 designation focus on the recall of information, such as definitions and terms, and simple procedures. Items with a DOK 2 designation require students to make decisions, solve routine problems, perform calculations, or recognize patterns. Items with a DOK 3 designation feature higher-order cognitive tasks. These DOK 3 tasks include but are not limited to: critiquing a statement and forming a conclusion; explaining, justifying, or proving a statement; or approaching abstract, complex, open-ended, and non-routine problems. Each grade’s blueprint contains information about the number of points of opportunity students will encounter at each DOK level.

Table 1: Math Descriptors – Applying Depth of Knowledge Levels for Mathematics (Webb, 2002) & NAEP 2002 Mathematics Levels of Complexity (M. Petit, Center for Assessment 2003, K. Hess, Center for Assessment, updated 2006)

<table>
<thead>
<tr>
<th>Level 1 Recall</th>
<th>Level 2 Skills/Concepts</th>
<th>Level 3 Strategic Thinking</th>
<th>Level 4 Extended Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Recall, observe, or recognize a fact, definition, term, or property</td>
<td>a. Classify plane and three-dimensional figures</td>
<td>a. Interpret information from a complex graph</td>
<td>a. Relate mathematical concepts to other content areas</td>
</tr>
<tr>
<td>b. Apply/compute a well-known algorithm (e.g., sum, quotient)</td>
<td>b. Interpret information from a simple graph</td>
<td>b. Explain thinking when more than one response is possible</td>
<td>b. Relate mathematical concepts to real-world applications in new situations</td>
</tr>
<tr>
<td>c. Apply a formula</td>
<td>c. Use models to represent mathematical concepts</td>
<td>c. Make and/or justify conjectures</td>
<td>c. Apply a mathematical model to illuminate a problem, situation</td>
</tr>
<tr>
<td>d. Determine the area or perimeter of rectangles or triangles given a drawing and labels</td>
<td>d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts</td>
<td>d. Use evidence to develop logical arguments for a concept</td>
<td>d. Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results</td>
</tr>
<tr>
<td>e. Identify a plane or three-dimensional figure</td>
<td>e. Compare and/or contrast figures or statements</td>
<td>e. Use concepts to solve non-routine problems</td>
<td>e. Design a mathematical model to inform and solve a practical or abstract situation</td>
</tr>
<tr>
<td>f. Measure</td>
<td>f. Construct 2-dimensional patterns for 3-dimensional models, such as cylinders and cones</td>
<td>f. Perform procedures with multiple steps and multiple decision points</td>
<td>f. Develop generalizations of the results obtained and the strategies used and apply them to new problem situations</td>
</tr>
<tr>
<td>g. Perform a specified or routine procedure (e.g., apply rules for rounding)</td>
<td>g. Provide justifications for steps in a solution process</td>
<td>g. Generalize a pattern</td>
<td></td>
</tr>
<tr>
<td>h. Evaluate an expression</td>
<td>i. Solve a one-step word problem</td>
<td>h. Extend a pattern</td>
<td></td>
</tr>
<tr>
<td>i. Solve a one-step word problem</td>
<td>j. Retrieve information from a table or graph</td>
<td></td>
<td></td>
</tr>
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</table>

Table 1 continued on next page.
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<th>Level 2 Skills/Concepts</th>
<th>Level 3 Strategic Thinking</th>
<th>Level 4 Extended Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>k. Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures</td>
<td>i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps</td>
<td>k. Solve a multiple-step problem and provide support with a mathematical explanation that justifies the answer</td>
<td>g. Apply one approach among many to solve problems</td>
</tr>
<tr>
<td>l. Locate numbers on a number line, or points on a coordinate grid</td>
<td>j. Translate between tables, graphs, words and symbolic notation</td>
<td>l. Solve 2-step linear equations/inequalities in one variable over the rational numbers, interpret solution(s) in the original context, and verify reasonableness of results</td>
<td>h. Apply understanding in a novel way, providing an argument/justification for the application</td>
</tr>
<tr>
<td>m. Solve linear equations</td>
<td>k. Make direct translations between problem situations and symbolic notation</td>
<td>m. Translate between a problem situation and symbolic notation that is not a direct translation</td>
<td></td>
</tr>
<tr>
<td>n. Represent math relationships in words, pictures, or symbols</td>
<td>l. Select a procedure according to criteria and perform it</td>
<td>n. Formulate an original problem, given a situation</td>
<td></td>
</tr>
<tr>
<td>o. Read, write, and compare decimals in scientific notation</td>
<td>m. Specify and explain relationships between facts, terms, properties, or operations</td>
<td>o. Analyze the similarities and differences between procedures</td>
<td></td>
</tr>
<tr>
<td></td>
<td>n. Compare, classify, organize, estimate, or order data</td>
<td>p. Draw conclusion from observations or data, citing evidence</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Level 4 involves such things as complex restructuring of data or establishing and evaluating criteria to solve problems.
Question 1

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.

What is the perimeter, in feet (ft), of Todd’s sandbox?

Points Possible: 1

Content Cluster: Understand the relationships between lengths, area, and volumes.

Content Standard: When figures are similar, understand and apply the fact that when a figure is scaled by a factor of $k$, the effect on lengths, areas, and volumes is that they are multiplied by $k$, $k^2$, and $k^3$, respectively. (G.GMD.6)

Depth of Knowledge: Level 2
b. Interpret information from a simple graph
e. Compare and/or contrast figures or statements
l. Select a procedure according to criteria and perform it
Scoring Guidelines

Exemplar Response

- 20.8 ft.

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- the correct perimeter (1 point).
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Question 1

Sample Responses
Sample Response: 1 point

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.

What is the perimeter, in feet (ft), of Todd’s sandbox?

20.8 ft
Notes on Scoring

This response earns full credit (1 point) because it shows a correctly identified perimeter of Todd's sandbox.

By the criterion of similar figures, Todd’s sandbox is similar to the park sandbox because the boxes are triangles with three pairs of corresponding congruent angles. Since corresponding sides in similar triangles are proportional, the scale factor is \( \frac{8}{20} \) or \( \frac{2}{5} \). The scale factor can be used either to find the missing side lengths of Todd’s sandbox and then to find its perimeter, or to find a perimeter of the park sandbox and then multiply it by a scale factor to find the perimeter of Todd’s sandbox.

The missing side lengths of Todd’s sandbox are \( 14 \cdot \frac{2}{5} = 5.6 \text{ ft} \) and \( 18 \cdot \frac{2}{5} = 7.2 \text{ ft} \). The perimeter of Todd’s sandbox is the sum of three side lengths, or \( 8 + 5.6 + 7.2 = 20.8 \text{ ft} \).

The perimeter of the park sandbox is the sum of three side lengths, or \( 20 + 14 + 18 = 52 \text{ ft} \). The perimeter of Todd’s sandbox is \( 52 \cdot \frac{2}{5} = 20.8 \text{ ft} \).
Sample Response: 1 point

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.

![Diagram of sandboxes]

What is the perimeter, in feet (ft), of Todd’s sandbox?

\[ 20 \frac{4}{5} \text{ ft} \]

Notes on Scoring

This response earns full credit (1 point) because it shows a correctly identified perimeter of Todd's sandbox.

The student provides the correct perimeter in an equivalent form.
Sample Response: 0 points

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.

What is the perimeter, in feet (ft), of Todd’s sandbox?

16 ft

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly identified perimeter of Todd’s Sandbox.

The student may estimate the scale factor as 2, multiply the length of 8 ft for Todd’s sandbox by 2 and then incorrectly use 16 ft as a perimeter of Todd’s sandbox.
Sample Response: 0 points

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.

What is the perimeter, in feet (ft), of Todd’s sandbox?

24 ft

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly identified perimeter of Todd’s sandbox.

The student may estimate the scale factor as 2 and use it to find missing side lengths of Todd’s sandbox as $\frac{14}{2} = 7$ ft and $\frac{18}{2} = 9$ ft. Then, the student may find a perimeter by adding the three side lengths, $8 + 7 + 9 = 24$ ft.
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Question 7

Question and Scoring Guidelines
Question 7

To estimate the area of the circle, Henry divides a circle of radius $r$ into $n$ triangles, as shown, and uses the expression $\frac{h}{2} (b_1 + b_2 + \ldots + b_n)$ to estimate the area of the circle. In the expression, variables $b_1, b_2, \ldots, b_n$ represent the base lengths of each triangle and $h$ represents the height of each triangle.

Henry claims that the more triangles the circle is divided into, the closer the estimated area will be to the actual area.

Which statement about Henry’s claim is accurate?

A. His claim is accurate because as $n$ gets larger, the value of $h$ gets closer to the value of $r$ and the value of $(b_1 + b_2 + \ldots + b_n)$ approaches $2\pi r$.

B. His claim is accurate because as $n$ gets larger, the value of $h$ gets closer to the value of $2r$ and the value of $(b_1 + b_2 + \ldots + b_n)$ approaches $\pi r$.

C. His claim is inaccurate because as $n$ gets larger, the value of $h$ gets closer to the value of $r$ and the value of $(b_1 + b_2 + \ldots + b_n)$ deviates from $2\pi r$.

D. His claim is inaccurate because as $n$ gets larger, the value of $h$ gets closer to the value of $2r$ and the value of $(b_1 + b_2 + \ldots + b_n)$ deviates from $\pi r$. 


Points Possible: 1

Content Cluster: Explain volume formulas and use them to solve problems.

Content Standard: Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments. (G.GMD.1)

Depth of Knowledge: Level 3
  a. Interpret information from a complex graph
  e. Use concepts to solve non-routine problems
  m. Translate between a problem situation and symbolic notation that is not a direct translation
Scoring Guidelines

Rationale for Option A: **Key** – The student correctly determines that the more triangles the circle is divided into, the closer the estimated area is to the actual area of a circle. The student correctly identifies the value of \( h \) as \( r \) and correctly realizes that as \( n \) gets larger, the base of each triangle approaches the arc of the circle and the sum of the base lengths of the triangles approaches the circumference of the circle.

Rationale for Option B: This is incorrect. The student may correctly recognize that the more triangles the circle is divided into, the closer the estimated area is to the actual area of a circle but may incorrectly think that the value of \( h \) is \( 2r \). The student may have correctly realized that as \( n \) gets larger, the base of each triangle approaches the arc of the circle but incorrectly think that the sum of the base lengths of the triangles approaches a value equal to half the circumference of the circle, by confusing the height of the triangles for the diameter of the circle.

Rationale for Option C: This is incorrect. The student may incorrectly assume that the more triangles the circle is divided into, the further the estimated area is from the actual area of a circle. The student may correctly identify the value of \( h \) as \( r \) but incorrectly determine that as \( n \) gets larger, the sum of the base lengths of the triangles deviates from the value equal to the circumference of the circle instead of approaching that value.

Rationale for Option D: This is incorrect. The student may incorrectly assume that the more triangles the circle is divided into, the further the estimated area is from the actual area. The student may incorrectly think that the value of \( h \) is \( 2r \), by confusing the height of the triangles for the diameter of the circle, and incorrectly determine that as \( n \) gets larger, the sum of the base lengths of the triangles deviates from the value equal to the circumference of the circle instead of approaching that value.
To estimate the area of the circle, Henry divides a circle of radius $r$ into $n$ triangles, as shown, and uses the expression $\frac{h}{2} \left(b_1 + b_2 + \ldots + b_n\right)$ to estimate the area of the circle. In the expression, variables $b_1, b_2, \ldots, b_n$ represent the base lengths of each triangle and $h$ represents the height of each triangle.

Henry claims that the more triangles the circle is divided into, the closer the estimated area will be to the actual area.

Which statement about Henry’s claim is accurate?

- His claim is accurate because as $n$ gets larger, the value of $h$ gets closer to the value of $r$ and the value of $\left(b_1 + b_2 + \ldots + b_n\right)$ approaches $2\pi r$.
- His claim is accurate because as $n$ gets larger, the value of $h$ gets closer to the value of $2r$ and the value of $\left(b_1 + b_2 + \ldots + b_n\right)$ approaches $\pi r$.
- His claim is inaccurate because as $n$ gets larger, the value of $h$ gets closer to the value of $r$ and the value of $\left(b_1 + b_2 + \ldots + b_n\right)$ deviates from $2\pi r$.
- His claim is inaccurate because as $n$ gets larger, the value of $h$ gets closer to the value of $2r$ and the value of $\left(b_1 + b_2 + \ldots + b_n\right)$ deviates from $\pi r$. 
Integrated Math II
Spring 2019 Item Release

Question 9

Question and Scoring Guidelines
Question 9

An expression is given, where \( x, y, \) and \( z \) represent positive real numbers.

\[
\frac{\frac{1}{y} + z}{x}
\]

Select the boxes to indicate whether increasing each variable causes the value of the expression to increase, decrease, or stay the same.

<table>
<thead>
<tr>
<th>Increasing ( x ), keeping ( y ) and ( z ) constant</th>
<th>Value increases</th>
<th>Value decreases</th>
<th>Value stays the same</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>Increasing ( y ), keeping ( x ) and ( z ) constant</td>
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<td></td>
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</tr>
<tr>
<td>Increasing ( z ), keeping ( x ) and ( y ) constant</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Points Possible: 1

Content Cluster: Interpret the structure of expressions.

Content Standard: Interpret expressions that represent a quantity in terms of its context. \((A.SSE.1)\)

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Depth of Knowledge: Level 2
e. Compare and/or contrast figures or statements
j. Translate between tables, graphs, words and symbolic notation
m. Specify and explain relationships between facts, terms, properties, or operations
### Scoring Guidelines

**Exemplar Response**

<table>
<thead>
<tr>
<th></th>
<th>Value increases</th>
<th>Value decreases</th>
<th>Value stays the same</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing $x$, keeping $y$ and $z$ constant</td>
<td>☐</td>
<td>✔</td>
<td>☐</td>
</tr>
<tr>
<td>Increasing $y$, keeping $x$ and $z$ constant</td>
<td>☐</td>
<td>✔</td>
<td>☐</td>
</tr>
<tr>
<td>Increasing $z$, keeping $x$ and $y$ constant</td>
<td>✔</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

**Other Correct Responses**

- N/A

For this item, a full-credit response includes

- the correctly completed table (1 point).
Sample Response: 1 point

An expression is given, where \( x, y, \) and \( z \) represent positive real numbers.

\[
\frac{1}{y} + \frac{z}{x}
\]

Select the boxes to indicate whether increasing each variable causes the value of the expression to increase, decrease, or stay the same.

<table>
<thead>
<tr>
<th></th>
<th>Value increases</th>
<th>Value decreases</th>
<th>Value stays the same</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing ( x ), keeping ( y ) and ( z ) constant</td>
<td>( \square )</td>
<td>( \checkmark )</td>
<td>( \square )</td>
</tr>
<tr>
<td>Increasing ( y ), keeping ( x ) and ( z ) constant</td>
<td>( \square )</td>
<td>( \checkmark )</td>
<td>( \square )</td>
</tr>
<tr>
<td>Increasing ( z ), keeping ( x ) and ( y ) constant</td>
<td>( \checkmark )</td>
<td>( \square )</td>
<td>( \square )</td>
</tr>
</tbody>
</table>
Notes on Scoring

This response earns full credit (1 point) because it shows a correctly completed table.

This question requires students to determine the impact of the changes in the certain variable onto the entire complex expression. Students should recognize that in the fractions where the numerator is a constant value and the denominator increases, the value of the fraction decreases. For example, for the fraction $\frac{1}{x}$, when $x = 5$, the value of the fraction is $\frac{1}{5}$; when $x = 6$, the value of the fraction is $\frac{1}{6}$; when $x = 7$, the value of the fraction is $\frac{1}{7}$, and so on. With the increase of $x$, the fraction’s value decreases. So, in this situation, if $y$ and $z$ are constant, then $\frac{1}{y}$ is also a constant and the entire numerator, ($\frac{1}{y} + z$), is a constant too. Since the denominator, $x$, increases, the entire expression decreases. The checkmark goes in the second column of the first row.

In the case where a fraction has a constant denominator value and the numerator increases, the value of the fraction increases too. For example, for the fraction $\frac{z}{10}$, when $z = 1$, the value of the fraction is $\frac{1}{10}$; when $z = 2$, the value of the fraction is $\frac{2}{10}$; when $z = 5$, the value of the fraction is $\frac{5}{10}$; and so on. With the increase of $z$, the fraction’s value increases. If $z$ increases and $y$ and $x$ are constant, then $\frac{1}{y}$ is a constant and the entire numerator, ($\frac{1}{y} + z$), increases. Since the denominator, $x$, is a constant, the entire expression increases with the increase of the numerator. The checkmark goes in the first column of the third row.

(continued on next page)
Notes on Scoring

Therefore, in the given situation, if $y$ increases and $z$ and $x$ remain constant, then $\frac{1}{y}$ decreases and the entire numerator, $\left(\frac{1}{y} + z\right)$, decreases too. Since the denominator, $x$, is a constant, the entire expression decreases with the decrease of the numerator. The checkmark goes in the second column of the second row.
Sample Response: 0 points

An expression is given, where $x$, $y$, and $z$ represent positive real numbers.

$$\frac{1}{y} + \frac{z}{x}$$

Select the boxes to indicate whether increasing each variable causes the value of the expression to increase, decrease, or stay the same.

<table>
<thead>
<tr>
<th>Increasing $x$, keeping $y$ and $z$ constant</th>
<th>Value increases</th>
<th>Value decreases</th>
<th>Value stays the same</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\checkmark$</td>
<td></td>
</tr>
<tr>
<td>Increasing $y$, keeping $x$ and $z$ constant</td>
<td></td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>Increasing $z$, keeping $x$ and $y$ constant</td>
<td>$\checkmark$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed table.

The student incorrectly places the check mark in the third column of the second row because the student may incorrectly think that because $y$ increases, $y$ approaches to the infinity, $\frac{1}{y}$ approaches to zero, the value of the entire numerator $(\frac{1}{y} + z)$ stays the same because $z$ is a constant and a sum of a constant and a zero is a constant. When the constant is divided by another constant, $x$, the value of the entire fraction stays the same.

The student does not realize that in the given situation, when $y$ increases (note: it has not been indicated that $y$ approaches to the infinity) and $z$ and $x$ remain constant, then $\frac{1}{y}$ decreases and the entire numerator, $(\frac{1}{y} + z)$, decreases too. Since the denominator, $x$, is a constant, the entire expression decreases with the decease of the numerator. The checkmark should go in the second column of the second row.
Sample Response: 0 points

An expression is given, where $x$, $y$, and $z$ represent positive real numbers.

$$\frac{\frac{1}{y} + z}{x}$$

Select the boxes to indicate whether increasing each variable causes the value of the expression to increase, decrease, or stay the same.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Increasing $x$, keeping $y$ and $z$ constant</td>
<td></td>
<td>⬜</td>
<td>□</td>
</tr>
<tr>
<td>Increasing $y$, keeping $x$ and $z$ constant</td>
<td>⬜</td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>Increasing $z$, keeping $x$ and $y$ constant</td>
<td></td>
<td></td>
<td>⬜</td>
</tr>
</tbody>
</table>

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed table. The student incorrectly places the check mark in the first column of the second row because the student may incorrectly think that the increase of $y$ would increase $\frac{1}{y}$ and because $z$ and $x$ are constants, the entire complex fraction increases.

Also, the student incorrectly places the check mark in the third column of the third row because the student may incorrectly think that since $z$ increases but everything else is constant the value of the entire complex fraction stays the same.
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Question 12

Question and Scoring Guidelines
Several features of a quadratic function over $x$ are shown.

- $y$ has a minimum value of 3
- $y$-intercept at $(0, 5)$
- no $x$-intercepts

Select all of the graphs and tables that illustrate all of the given features.
Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context.

Content Standard: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. b. Focus on linear, quadratic, and exponential functions. (A1, M2) (F.IF.4)

Depth of Knowledge: Level 2  
b. Interpret information from a simple graph  
j. Translate between tables, graphs, words and symbolic notation that is not a direct translation
Scoring Guidelines

Rationale for First Option: Key – The student correctly chooses the graph that is opened up, above the x-axis, without x-intercepts, with the vertex (lowest point) at (1,3) and intersecting the y-axis at y=5.

Rationale for Second Option: This is incorrect. The student may choose the graph that has the y-intercept at (0,5), but the graph has a maximum value rather than minimum, and has two x-intercepts.

Rationale for Third Option: This is incorrect. The student may choose the graph that has the correct y-intercept at (0,5) and no x-intercepts but has a minimum value at y=5 instead of y=3.

Rationale for Fourth Option: This is incorrect. The student may choose a table that has the y-intercept at (0,5) (since y=5 when x=0) but shows values of the function less than 3, and also must have an x-intercept between x=1 and x=2 because it goes from positive to negative in this interval.

Rationale for Fifth Option: This is incorrect. The student may flip the x- and y-coordinates in the table, since this would represent a quadratic relation in y; where x has a minimum value of 3, there is an x-intercept at (5,0) and there are no y-intercepts.

Rationale for Sixth Option: Key – The student correctly identified that the values in the table are symmetric about the point (1,3) and all of the values in the table are greater than 3, so y has a minimum value of 3. Also, since y=5 when x=0, the quadratic function described by this table has a y-intercept at (0,5). Finally, since y has a minimum value of 3, the graph of the function does not cross the x-axis. So, there are no x-intercepts.
Several features of a quadratic function over \( x \) are shown.

- \( y \) has a minimum value of 3
- \( y \)-intercept at \((0, 5)\)
- no \( x \)-intercepts

Select all of the graphs and tables that illustrate all of the given features.

- Table 1: \( x \) y
  - \((-1, 7)\)
  - \((0, 5)\)
  - \((1, 1)\)
  - \((2, -3)\)

- Table 2: \( x \) y
  - \((11, -1)\)
  - \((5, 0)\)
  - \((3, 1)\)
  - \((5, 2)\)

- Table 3: \( x \) y
  - \((-1, 11)\)
  - \((0, 5)\)
  - \((1, 3)\)
  - \((2, 5)\)
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Question 13

Question and Scoring Guidelines
Question 13

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

Points Possible: 1

Content Cluster: Apply geometric concepts in modeling situations.

Content Standard: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (G.MG.3)

Depth of Knowledge: Level 2
d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts
k. Make direct translations between problem situations and symbolic notation
l. Select a procedure according to criteria and perform it
Scoring Guidelines

Exemplar Response

- 7.98 centimeters

Other Correct Responses

- any equivalent value from 7.977 to 8.923

For this item, a full-credit response includes:

- a correct radius (1 point).
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Question 13

Sample Responses
Sample Response: 1 point

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

7.98 centimeters
To design a cylinder with the fixed height of 10 cm and the volume that is not larger than 2,500 cubic centimeters but not smaller than 2,000 cubic centimeters, the company needs to determine the largest and the smallest radius of the base. The borderline (minimum and maximum) values for the radius can be found by using the formula for the volume of the cylinder, \( V = \pi r^2 h \), where \( r \) is the radius of the base and \( h \) is the height of the cylinder. The equation for the volume of the cylinder with \( V = 2000 \) is \( \pi r^2 \cdot 10 = 2000 \) and the equation for the volume of the cylinder with \( V = 2500 \) is \( \pi r^2 10 = 2500 \). Solve the two equations to obtain border line values for the radius.

\[
\begin{align*}
\pi r^2 10 &= 2000 \quad &\pi r^2 10 &= 2500 \\
\pi r^2 &= 200 \quad &\pi r^2 &= 250 \\
r^2 &= 200 \div \frac{22}{7} \text{ (using } \frac{22}{7} \text{ for } \pi) \quad &r^2 &\approx \frac{250}{3.14} \text{ (using 3.14 for } \pi) \\
r &\approx \pm 7.977 \quad &r &\approx \pm 8.923
\end{align*}
\]

Since the radius of the cylinder can only be a positive number, only positive square roots must be considered. Depending on a value used for \( \pi \), any real number between 7.977 and 8.923 is acceptable for the radius of the cylinder to keep the volume not larger than 2,500 cubic centimeters and not smaller than 2,000 cubic centimeters.
Sample Response: 1 point

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

8 centimeters

Notes on Scoring

This response earns full credit (1 point) because it shows a correctly identified value for the radius of the cylindrical object.

The student identifies a value that is consistent with the given dimensions and the range of the acceptable responses.
Sample Response: 0 points

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

62.68 centimeters

Notes on Scoring

This response earns no credit (0 points) because it shows a value for the radius of the cylindrical object that is inconsistent with the given dimensions and the range of the acceptable responses.

The student may create the correct equations for the borderline values of the radius using the volume of a cylinder formula but forget to apply a square root to both sides of the equations and round the answer down.
Sample Response: 0 points

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

13.82 centimeters
Notes on Scoring

This response earns no credit (0 points) because it shows a value for the radius of the cylindrical object that is inconsistent with the given dimensions and the range of the acceptable responses.

The student may create the equations for the borderline values of the radius using the formula for the volume of a cone instead of a cylinder and then accurately solve them for $r$:

\[ \frac{1}{3} \pi r^2 10 = 2000 \quad \text{and} \quad \frac{1}{3} \pi r^2 10 = 2500 \]
\[ \pi r^2 = 600 \quad \text{and} \quad \pi r^2 = 750 \]
\[ r^2 \approx 190.9859 \quad \text{and} \quad r^2 \approx 238.7324 \]
\[ r \approx \pm 13.8198 \quad \text{and} \quad r \approx \pm 15.45097 \]
Integrated Math II
Spring 2019 Item Release

Question 16

Question and Scoring Guidelines
Question 16

Two functions, \( f \) and \( g \), are given.

\[
\begin{align*}
f(x) &= x^2 \\
g(x) &= 2^x
\end{align*}
\]

Which statement is true about the outputs of the two functions?

A. \( g(x) > f(x) \) for all values of \( x \).
B. \( f(x) > g(x) \) for all values of \( x \).
C. \( f(x) > g(x) \) for all values of \( x > 2 \).
D. \( g(x) > f(x) \) for all values of \( x > 4 \).

Points Possible: 1

Content Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

Content Standard: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (A1, M2) (F.LE.3)

Depth of Knowledge: Level 2

- e. Compare and/or contrast figures or statements
- j. Translate between tables, graphs, words and symbolic notation
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may think that since in general, exponential functions grow faster than quadratic functions, the output values of exponential functions are always larger than output values of quadratic functions. However, it is only as $x$ gets significantly large that exponential functions grow at a faster rate than quadratic functions.

Rationale for Option B: This is incorrect. The student may think that since for some values of $x$ the output values for the quadratic function are larger than for the exponential function, it is also true for all values of $x$. However, as $x$ gets significantly large, exponential functions grow at a faster rate than quadratic functions.

Rationale for Option C: This is incorrect. The student may note that the outputs of the two functions are equal at $x = 2$ and for some values of $x > 2$ the output values of $f(x)$ are greater than the output values of $g(x)$. However, the student may fail to notice that at $x = 4$ the outputs of the functions are equal again and for $x > 4$ the outputs of exponential function $g$ are greater than output values of the quadratic function $f$.

Rationale for Option D: Key – The student correctly identifies that the outputs of the two functions are equal at $x = 2$ and $x = 4$. The student also notes that the exponential function $g$ has a greater output value than the quadratic function $f$ for $2 < x < 4$ and for all values of $x$ larger than 4, which makes this option true.
Sample Response: 1 point

Two functions, $f$ and $g$, are given.

$$f(x) = x^2$$
$$g(x) = 2^x$$

Which statement is true about the outputs of the two functions?

- **A** $g(x) > f(x)$ for all values of $x$.
- **B** $f(x) > g(x)$ for all values of $x$.
- **C** $f(x) > g(x)$ for all values of $x > 2$.
- $g(x) > f(x)$ for all values of $x > 4$. 


Question 20

A circle with center L contains points J and K. Circle L is dilated by a factor of 2, resulting in a new circle with center P. Points M and N are on circle P such that central angle MPN has the same measure as central angle JLK.

Which statement correctly identifies the relationship between the arc length of JK and the arc length of MN?

A. The arc length of JK is half the arc length of MN.
B. The arc length of MN is half the arc length of JK.
C. The arc length of JK is a quarter of the arc length of MN.
D. The arc length of MN is a quarter of the arc length of JK.

Points Possible: 1

Content Cluster: Find arc lengths and areas of sectors of circles.

Content Standard: Find arc lengths and areas of sectors of circles. (G.C.5)
a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.

Depth of Knowledge: Level 2
c. Use models to represent mathematical concepts
e. Compare and/or contrast figures or statements
Scoring Guidelines

Rationale for Option A: **Key** – The student understands that each arc length is proportional to the radius \((\text{Arc Length} = (\text{measure of central angle}) \times (\text{radius}))\). So, after the dilation by a factor 2 the radius of circle \(P\) is 2 times the radius of circle \(L\) and for the same central angle the longer arc is 2 times as long as the shorter arc length, and the shorter arc length is \(1/2\) that of the longer arc length.

Rationale for Option B: This is incorrect. The student may understand that arc length is proportional to the radius but switched the order of the proportion.

Rationale for Option C: This is incorrect. The student may confuse the arc length with the area of a sector and think that if the radius of a circle \(L\) is multiplied by 2, then the area of a sector is going to be multiplied by \(2^2\) or 4. Therefore, if \(4JK = MN\), then \(JK = \frac{1}{4}MN\).

Rationale for Option D: This is incorrect. The student may confuse circle \(L\) with circle \(P\) and the arc length with the area of a sector. The student may incorrectly think that if the radius of a circle \(P\) is multiplied by 2, then the area of a sector is going to be multiplied by \(2^2\) or 4. Therefore, if \(4MN = JK\), then \(MN = \frac{1}{4}JK\).

Sample Response: 1 point

A circle with center \(L\) contains points \(J\) and \(K\). Circle \(L\) is dilated by a factor of 2, resulting in a new circle with center \(P\). Points \(M\) and \(N\) are on circle \(P\) such that central angle \(MPN\) has the same measure as central angle \(JLK\).

Which statement correctly identifies the relationship between the arc length of \(JK\) and the arc length of \(MN\)?

- The arc length of \(JK\) is half the arc length of \(MN\).
- The arc length of \(MN\) is half the arc length of \(JK\).
- The arc length of \(JK\) is a quarter of the arc length of \(MN\).
- The arc length of \(MN\) is a quarter of the arc length of \(JK\).
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Question 22

Question and Scoring Guidelines
Question 22

An expression is given.

\[(2x + 8)(5x - 7)\]

Which expression is equivalent to the given expression?

A. \[36x - 56\]
B. \[10x^2 - 56\]
C. \[10x^2 + 26x - 56\]
D. \[10x^2 + 54x + 56\]

Points Possible: 1

Content Cluster: Perform arithmetic operations on polynomials

Content Standard: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) (A.APR.1)

Depth of Knowledge: Level 1
b. Apply/compute a well-known algorithm (e.g., sum, quotient)
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may attempt to multiply each term of the first binomial by each term of the second binomial but make a mistake when multiplying $2x \cdot 5x$ to get $10x$ and then correctly combine $10x$ with the rest of the like terms, $10x – 14x + 40x – 56 = 36x – 56$.

Rationale for Option B: This is incorrect. The student may attempt to multiply only the first terms ($2x$ and $5x$) and the last terms ($8$ and $–7$) of the two binomials to get $2x \cdot 5x + 8 \cdot (–7) = 10x^2 – 56$ instead of multiplying each term of the first binomial by each term of the second binomial.

Rationale for Option C: **Key** – The student correctly multiplies each term of the first binomial by each term of the second binomial, and then combines like terms $(2x + 8)(5x – 7) = 2x \cdot 5x + 8 \cdot 5x + 2x \cdot (–7) + 8 \cdot (–7) = 10x^2 + 40x – 14x – 56 = 10x^2 + 26x – 56$.

Rationale for Option D: This is incorrect. The student may attempt to multiply the two binomials but may neglect to include the negative sign when multiplying $(2x + 8)$ by $(–7)$, finding the products of $2x \cdot (7)$ and $8 \cdot (7)$ instead of $2x \cdot (–7)$ and $8 \cdot (–7)$ to get $2x \cdot 5x + 8 \cdot 5x + 2x \cdot 7 + 8 \cdot 7 = 10x^2 + 40x + 14x + 56 = 10x^2 + 54x + 56$.

Sample Response: 1 point

```
An expression is given.

$(2x + 8)(5x – 7)$

Which expression is equivalent to the given expression?

A  $36x – 56$
B  $10x^2 – 56$
C  $10x^2 + 26x – 56$
D  $10x^2 + 54x + 56$
```
Question 23

Bryan records the number of hours he sleeps each night for several days and whether it is raining in the morning when he wakes up. Bryan concludes that these two events are independent:

- Bryan sleeps 8 or more hours.
- It is raining in the morning.

Based on Bryan’s conclusion, which statement must be true?

A. Bryan never sleeps 8 or more hours on days that it is not raining in the morning.
B. The probability that Bryan sleeps 8 or more hours is the same whether or not it is raining in the morning.
C. The probability that Bryan sleeps 8 or more hours is influenced by whether or not it is raining in the morning.
D. The probability that Bryan sleeps 8 or more hours is the same as the probability that it is raining in the morning.

Points Possible: 1

Content Cluster: Understand independence and conditional probability, and use them to interpret data.

Content Standard: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)

Depth of Knowledge: Level 2
e. Compare and/or contrast figures or statements
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly think that if two events are independent, then the negation of the first event and the negation of the second event must happen together, but it only means that the probability of one event is not influenced by the other event occurring.

Rationale for Option B: Key – The student correctly realizes that if two events are independent, then the probability of one event happening is not influenced by the other event occurring. If the two events are independent, then the probability that Bryan sleeps 8 or more hours is not influenced by whether it rains in the morning.

Rationale for Option C: This is incorrect. The student may switch the meanings of independent and dependent events, since the probability of one event happening is influenced by the other event occurring only when events are dependent.

Rationale for Option D: This is incorrect. The student may think that if two events are independent, then they have equal probability of occurring, but it only means that their probabilities are not influenced by the other event occurring.
Sample Response: 1 point

Bryan records the number of hours he sleeps each night for several days and whether it is raining in the morning when he wakes up. Bryan concludes that these two events are independent:

- Bryan sleeps 8 or more hours.
- It is raining in the morning.

Based on Bryan’s conclusion, which statement must be true?

A. Bryan never sleeps 8 or more hours on days that it is not raining in the morning.

B. The probability that Bryan sleeps 8 or more hours is the same whether or not it is raining in the morning.

C. The probability that Bryan sleeps 8 or more hours is influenced by whether or not it is raining in the morning.

D. The probability that Bryan sleeps 8 or more hours is the same as the probability that it is raining in the morning.
Integrated Math II
Spring 2019 Item Release

Question 24

Question and Scoring Guidelines
Question 24

Two triangles are shown, where $BD$ and $AE$ intersect at point $C$.

What is the perimeter, in centimeters (cm), of $\triangle ABC$?

\[ cm \]
Points Possible: 1

**Content Cluster:** Prove and apply theorems both formally and informally involving similarity using a variety of methods.

**Content Standard:** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. (G.SRT.5)

**Depth of Knowledge:** Level 2  
b. Interpret information from a simple graph  
d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts

### Scoring Guidelines

**Exemplar Response**

- 50 cm

**Other Correct Responses**

- any equivalent value

For this item, a full-credit response includes:

- the correct ordered pair (1 point).
Sample Response: 1 point

Two triangles are shown, where $BD$ and $AE$ intersect at point $C$.

What is the perimeter, in centimeters (cm), of $\triangle ABC$?

50 $cm$

1 2 3
4 5 6
7 8 9
0

58 (2019)
Notes on Scoring

This response earns full credit (1 point) because it shows a correct perimeter of the triangle.

There are several approaches to answer this question. One of them is to find the length of side $AB$ and then to calculate the perimeter of the triangle as the sum of three side lengths, or $P = AB + 17 + 18$.

Two shown triangles are congruent by the Side-Angle-Side (SAS) criterion because they have two pairs of congruent corresponding sides and a pair of congruent vertical angles between those sides. Since in congruent triangles pairs of corresponding side lengths are equal, then $AB = DE = 15$ cm. Therefore, the perimeter of triangle $ABC$ is $15 + 17 + 18 = 50$ cm.

Another approach is to establish a congruence of two shown triangles and then conclude that the perimeters of congruent triangles are equal, because the corresponding sides have equal lengths. The perimeter of triangle $CDE$ is $15 + 17 + 18 = 50$ cm, so the perimeter of triangle $ABC$ is also $50$ cm.
Two triangles are shown, where BD and AE intersect at point C.

What is the perimeter, in centimeters (cm), of \( \triangle ABC \)?

50.0 cm
Notes on Scoring

This response earns full credit (1 point) because it shows the correct perimeter of the triangle.

The student provides an equivalent value for the perimeter of the triangle.
Sample Response: 0 points

Two triangles are shown, where $BD$ and $AE$ intersect at point $C$.

What is the perimeter, in centimeters ($cm$), of $\triangle ABC$?

35 $cm$
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect perimeter of the triangle.

The student may find the sum of the two labeled sides of the triangle, ignoring that the perimeter is the sum of the three side lengths.
Two triangles are shown, where BD and AE intersect at point C.

What is the perimeter, in centimeters (cm), of ΔABC?

100 cm
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect perimeter of the triangle.

The student may find the sum of the perimeters of two shown triangles.
Question 26

A cylinder is sliced vertically along a dotted line, as shown.

Which two-dimensional shape is created from this cross section?

A  

B  

C  

D
**Scoring Guidelines**

**Rationale for Option A: Key** – The student correctly notes the result of a rotation about a vertical line through its center is a cylinder. Thus, a vertical slice of a cylinder along the dotted line leads to a rectangular cross section.

**Rationale for Option B:** This is incorrect. The student may think that since the sides of the cylinder are straight and the bases of the cylinder are circles, that the cross section has 2 straight sides and 2 curved sides. However, if this shape is rotated around a vertical line around its center, the result of the rotation is a cylinder-like figure with bowed out sides. Thus, this would be a vertical cross section for a cylinder-like figure with bowed out sides.

**Rationale for Option C:** This is incorrect. The student may think that every cross section of a cylinder is a circle, but instead, this is only true for horizontal slices parallel to the base.

**Rationale for Option D:** This is incorrect. The student may think that a cross section of a cylinder would have to be some sort of curved shape and also notice that the cylinder is taller than it is wide.
A cylinder is sliced vertically along a dotted line, as shown.

Which two-dimensional shape is created from this cross section?
Question 28

A function $f(x)$ is shown.

Which graph represents $f(x) - 2$?
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly think that $f(x) - 2$ would subtract 2 from the $x$-values and translate the graph two units to the left.

Rationale for Option B: This is incorrect. The student may incorrectly think that $f(x) - 2$ would add 2 to the $x$-values and translate the graph two units to the right.

Rationale for Option C: Key – The student correctly determines that $f(x) - 2$ would subtract 2 from the $y$-values and translate the graph two units down.

Rationale for Option D: This is incorrect. The student may incorrectly think that $f(x) - 2$ would add 2 to the $y$-values and translate the graph two units up.
A function $f(x)$ is shown.

Which graph represents $f(x) - 2$?

(A)  

(B)  

(C)  

(D)
Question 31

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

<table>
<thead>
<tr>
<th></th>
<th>Amusement Park</th>
<th>Zoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>11th Graders</td>
<td>32</td>
<td>18</td>
</tr>
<tr>
<td>12th Graders</td>
<td>24</td>
<td>26</td>
</tr>
</tbody>
</table>

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?
Points Possible: 1

Content Cluster: Understand independence and conditional probability, and use them to interpret data.

Content Standard: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)

Depth of Knowledge: Level 2
i. Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps
k. Make direct translations between problem situations and symbolic notation

Scoring Guidelines

Exemplar Response

• 0.36

Other Correct Responses

• any equivalent value

For this item, a full-credit response includes

• the correct value (1 point).
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Question 31

Sample Responses
Sample Response: 1 point

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

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<td>24</td>
<td>26</td>
</tr>
</tbody>
</table>

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

\[
\frac{18}{50}
\]
Notes on Scoring

This response earns full credit (1 point) because it shows the correct conditional probability of an 11th grade student preferring the zoo.

The situation requires the ability to interpret information summarized in a two-way table. The total number of 11th graders is 32 + 18 or 50 students. Out of these, 18 students prefer to go to the zoo. Therefore, the probability that the student prefers to go to the zoo, given that the student is an 11th grader is \( \frac{18}{50} \).
Sample Response: 1 point

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

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</tr>
<tr>
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<td>24</td>
<td>26</td>
</tr>
</tbody>
</table>

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

0.36
Notes on Scoring

This response earns full credit (1 point) because it shows the correct conditional probability of an 11th grade student preferring the zoo.

The situation requires the ability to interpret information summarized in a two-way table. The total number of 11th graders is 32 + 18 or 50 students. Out of these, 18 students prefer to go to the zoo. Therefore, the probability that the student prefers to go to the zoo, given that the student is an 11th grader is $\frac{18}{50}$, or 0.36.
Sample Response: 1 point

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

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</tr>
</tbody>
</table>

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

\[
\frac{9}{25}
\]

Notes on Scoring

This response earns full credit (1 point) because it shows the correct conditional probability of an 11th grade student preferring the zoo in the equivalent form.

The probability that the student prefers to go to the zoo, given that the student is an 11th grader is \(\frac{18}{50}\), or \(\frac{9}{25}\).
Sample Response: 0 points

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

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<td>24</td>
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</tr>
</tbody>
</table>

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

\[
\frac{9}{16}
\]
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect conditional probability of an 11th grade student preferring the zoo.

The student may incorrectly calculate the probability of an 11th grade student preferring the amusement park, \( \frac{18}{32} \) or \( \frac{9}{16} \), instead of calculating the probability of an 11th grade student preferring the zoo, or \( \frac{18}{50} \).
Sample Response: 0 points

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

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<tr>
<td>12th Graders</td>
<td>24</td>
<td>26</td>
</tr>
</tbody>
</table>

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

\[
\frac{18}{44}
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect conditional probability of an 11th grade student preferring the zoo.

Instead, the student may calculate the probability of an 11th grader preferring a class trip to the zoo out of all students who prefer to go to the zoo, or \( \frac{18}{18+26} = \frac{18}{44} \).
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Question 35

Question and Scoring Guidelines
Question 35

An equation is given, where \( x \geq 0 \).

\[ y = x^2 + 1 \]

Create an equivalent equation that is solved for \( x \) in terms of \( y \).

\[ x = \]

Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

Content Standard: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. c. Focus on formulas in which the variable of interest is linear or square. For example, rearrange the formula for the area of a circle \( A = \pi r^2 \) to highlight radius \( r \). (M2) (A.CED.4)

Depth of Knowledge: Level 2
d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts
Scoring Guidelines

Exemplar Response

- $x = \sqrt{y - 1}$

Other Correct Responses

- any equivalent equation

For this item, a full-credit response includes:

- a correct equation (1 point).
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Question 35

Sample Responses
Sample Response: 1 point

An equation is given, where \( x \geq 0 \).

\[ y = x^2 + 1 \]

Create an equivalent equation that is solved for \( x \) in terms of \( y \).

\[ x = \sqrt{y-1} \]
Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation expressing \( x \) in terms of \( y \).

To solve the equation for \( x \), rearrange a given equation \( y = x^2 + 1 \) to isolate a variable \( x \) by applying the same reasoning as used in solving a quadratic equation. First, use Addition Property of Equality (or Subtraction Property of Equality) to add \((-1)\) from both sides:

\[
y + (-1) = x^2 + 1 + (-1) \\
y - 1 = x^2
\]

Then, apply the square root to both sides of the equation to get:

\[
\sqrt{y - 1} = \sqrt{x^2}.
\]

Since \( x \geq 0 \), there is only one equivalent equation that solves for \( x \) in terms of \( y \).

\[
\sqrt{y - 1} = x
\]

Last, use the Reflexive Property of Equality to interchange the sides of the last equation to get \( x = \sqrt{y - 1} \).
Sample Response: 1 point

An equation is given, where $x \geq 0$.

$y = x^2 + 1$

Create an equivalent equation that is solved for $x$ in terms of $y$.

$x = (y - 1)^{0.5}$

Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation expressing $x$ in terms of $y$.

The student recognizes that the equation $x = \sqrt{y - 1}$ can be restated as $x = (y - 1)^{0.5}$. 
Sample Response: 0 points

An equation is given, where $x \geq 0$.

$y = x^2 + 1$

Create an equivalent equation that is solved for $x$ in terms of $y$.

$x = \frac{y - 1}{2}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect equation that is solved for $x$ in terms of $y$.

First, the student may use Addition Property of Equality (or Subtraction Property of Equality) to add $(-1)$ from both sides:

$y + (-1) = x^2 + 1 + (-1)$

$y - 1 = x^2$

Then, the student may incorrectly divide both sides of the equation by 2 to get $x = \frac{y - 1}{2}$ instead of applying the square root to both sides of the equation.
An equation is given, where \( x \geq 0 \).

\[ y = x^2 + 1 \]

Create an equivalent equation that is solved for \( x \) in terms of \( y \).

\[ x = \sqrt{y - 1} \]
Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect equation that is solved for \( x \) in terms of \( y \).

First, the student may use Addition Property of Equality (or Subtraction Property of Equality) to add \((-1)\) from both sides:

\[ y + (-1) = x^2 + 1 + (-1) \]
\[ y - 1 = x^2 \]

Next, the student may apply the square root to both sides of the equation to get

\[ \sqrt{y - 1} = \sqrt{x^2} \]

Since \( x \geq 0 \), there is only one equivalent equation that solves for \( x \) in terms of \( y \)

\[ \sqrt{y - 1} = x \]

Last, the student may incorrectly separate \( \sqrt{y - 1} \) into \( \sqrt{y} \) and \( \sqrt{1} \) to get \( \sqrt{y} - 1 = x \) or \( x = \sqrt{y} - 1 \).
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Question 37

Question and Scoring Guidelines
Question 37

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add \[ \phantom{0.50} \] and then \[ \phantom{0.50} \].

**Drop down choices**

- To find the probability that either Ralph or Dan will play in the tournament, first add
  - 0.50 and 0.48
  - 0.50 and 0.25
  - 0.48 and 0.25

- And then
  - subtract 0.25 from the sum.
  - multiply the sum by 0.48.
  - divide 0.50 by the sum.
  - subtract 0.50 from the sum.
  - multiply the sum by 0.25.
Points Possible: 1

Content Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Content Standard: Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model. (S.CP.7)

Depth of Knowledge: Level 2
  c. Use models to represent mathematical concepts
  l. Select a procedure according to criteria and perform it
  k. Make direct translations between problem situations and symbolic notation
Scoring Guidelines

Exemplar Response

- “0.50 and 0.48” and “subtract 0.25 from the sum”

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- the correctly completed statement (1 point).
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Question 37

Sample Responses
Sample Response: 1 point

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add 0.50 and 0.48 and then subtract 0.25 from the sum.

Notes on Scoring

This response earns full credit (1 point) because it shows the correctly completed statement using probabilities of events.

To find a probability of either Ralph or Dan playing in the tournament, use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where $P(A)$ is a probability of one event, $P(B)$ is the probability of another event and $P(A \text{ and } B)$ is the probability of both events happening at the same time.

If there is a 50% chance that Ralph plays in the tournament, then the probability of him playing is 0.5, or $P(A) = 0.5$. The probability that Dan plays is 0.48, or $P(B) = 0.48$, and the probability that both Ralph and Dan play is 0.25, or $P(A \text{ and } B)$. Following the Addition Rule, the probability that either Ralph or Dan play in the tournament, first add 0.50 and 0.48, and then subtract 0.25 from the sum.
Sample Response: 0 points

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add 0.50 and 0.48 and then multiply the sum by 0.25.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed statement using probabilities of events.

To find a probability of either Ralph or Dan playing in the tournament, the student may incorrectly use the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), by first adding \( P(A) + P(B) \), or 0.5 and 0.48, and then multiplying the result by \( P(A) + P(B) \), or 0.25, instead of subtracting 0.25.
Sample Response: 0 points

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add \(0.48\) and \(0.25\), and then subtract \(0.50\) from the sum.

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed statement using probabilities of events.

The student may attempt to use the Addition Rule, 
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \], but incorrectly use two smaller probabilities, 0.48 and 0.25, to replace \(P(A) + P(B)\) and one larger probability, 0.50, to replace \(P(A \text{ and } B)\), so that the statement reads “first add 0.48 and 0.25, and then subtract 0.50 from the sum”.

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Question 38

Question and Scoring Guidelines
Question 38

A quadratic equation with variable \( x \) is shown.

\[ x^2 + (2b - 3a)x - 6ab = 0 \]

What are the solutions to this equation in terms of the constants \( a \) and \( b \)?

\[ x = \quad \]

\[ x = \quad \]
Points Possible: 2

Content Cluster: Solve equations and inequalities in one variable.

Content Standard: Solve quadratic equations in one variable. (A.REI.4)
b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for \(x^2 = 49\); taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

Depth of Knowledge: Level 3
e. Use concepts to solve non-routine problems
b. Explain thinking when more than one response is possible

Scoring Guidelines

Exemplar Response

- \(x = 3a\)
- \(x = -2b\)

Other Correct Responses

- any equivalent expressions

For this item, a full-credit response includes:

- one correct solution (1 point)
  AND
- a second correct solution (1 point).
Integrated Math II
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Question 38

Sample Responses
Sample Response: 2 points

A quadratic equation with variable $x$ is shown.

$$x^2 + (2b - 3a)x - 6ab = 0$$

What are the solutions to this equation in terms of the constants $a$ and $b$?

$$x = 3a$$

$$x = -2b$$
Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions to the quadratic equation in terms of constants $a$ and $b$.

There are several ways to answer this question. One of them is to use factoring and the Zero-Product Property. A quadratic equation in the standard form $Ax^2 + Bx + C = 0$, where $A \neq 0$, must be factored before the Zero-Product Property is applied.

Since the given quadratic equation $x^2 + (2b - 3a)x - 6ab = 0$ has $A=1$, focus on the case when a standard form of the quadratic equation is $1x^2 + Bx + C = 0$.

To factor $x^2 + Bx + C$, use the Distributive Property in reverse. The factorization would look like $(x - x_1)(x - x_2) = 0$, where $(-x_1)$ and $(-x_2)$ are two numbers whose product is $C$ and the sum is $B$. The pair of numbers satisfying these two conditions are numbers to be used in factorization.

The Zero-Product Property states that a product of two factors is zero, if and only if each or both factors are zero. This translates to two separate linear equations as $x - x_1 = 0$ or $x - x_2 = 0$, from where, the solutions are $x = x_1$ and $x = x_2$.

Apply the same idea to the equation $x^2 + (2b - 3a)x - 6ab = 0$. Two numbers used in a factorization, $(-x_1) + (-x_2)$, are such that:

$(-x_1) + (-x_2) = 2b - 3a$ and $-x_1 \cdot (-x_2) = -6ab$

$x_1 + x_2 = 3a - 2b$ \hspace{1cm} $x_1 \cdot x_2 = -6ab$

Since the solutions $x_1$ and $x_2$ must satisfy two conditions, the solutions are $3a$ and $(-2b)$. 
Sample Response: 2 points

A quadratic equation with variable $x$ is shown.

$$x^2 + (2b - 3a)x - 6ab = 0$$

What are the solutions to this equation in terms of the constants $a$ and $b$?

$$x = \frac{6}{2a}$$

$$x = \frac{-4}{2b}$$

Notes on Scoring

This response earns full credit (2 points) because it shows two correct solutions to the quadratic equation in terms of constants $a$ and $b$.

The student provides equivalent solutions to the quadratic equation because $\frac{6}{2a} = 3a$ and $\frac{-4}{2b} = -2b$. 
Sample Response: 1 point

A quadratic equation with variable $x$ is shown.

$$x^2 + (2b - 3a)x - 6ab = 0$$

What are the solutions to this equation in terms of the constants $a$ and $b$?

$x = \boxed{3a}$

$x = \boxed{2b}$

Notes on Scoring

This response earns partial credit (1 point) because it shows one correct solution, $3a$, and one incorrect solution, $2b$, to the quadratic equation in terms of constants $a$ and $b$.

With these solutions, the quadratic equation in the factored form is $(x - 3a)(x - 2b) = 0$. When distributed, the equation is $x^2 - 2bx - 3ax + 6ab = 0$.

When $x$ is factored from the second and the third term of this equation, the result is $x^2 - (2b + 3a)x + 6ab = 0$, which is not equivalent to $x^2 + (2b - 3a)x - 6ab = 0$. 
Sample Response: 1 point

A quadratic equation with variable $x$ is shown.

$$x^2 + (2b - 3a)x - 6ab = 0$$

What are the solutions to this equation in terms of the constants $a$ and $b$?

$$x = \boxed{-3a}$$

$$x = \boxed{-2b}$$

Notes on Scoring

This response earns partial credit (1 point) because it shows one incorrect solution, $-3a$, and one correct solution, $-2b$, to the quadratic equation in terms of constants $a$ and $b$.

With these solutions, the quadratic equation in the factored form is $(x + 3a)(x + 2b) = 0$. When distributed, the equation is $x^2 + 2bx + 3ax + 6ab = 0$. When $x$ is factored from the second and third term, the equation is $x^2 + (2b + 3a)x + 6ab = 0$, which is not equivalent to $x^2 + (2b - 3a)x - 6ab = 0$. 
A quadratic equation with variable $x$ is shown.

$x^2 + (2b - 3a)x - 6ab = 0$

What are the solutions to this equation in terms of the constants $a$ and $b$?

$x = -3a$

$x = 2b$

Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect solutions.

With these solutions, the quadratic equation in the factored form is $(x + 3a)(x - 2b) = 0$. When distributed, the equation is $x^2 - 2bx + 3ax - 6ab = 0$. When $x$ is factored from the second and third term, the equation is $x^2 - (2b - 3a)x - 6ab = 0$, which is not equivalent to $x^2 + (2b - 3a)x - 6ab = 0$. 
Sample Response: 0 points

A quadratic equation with variable $x$ is shown.

$$x^2 + (2b - 3a)x - 6ab = 0$$

What are the solutions to this equation in terms of the constants $a$ and $b$?

$$x = 6a$$

$$x = 1b$$

Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect solutions.

With these solutions, the quadratic equation in the factored form is $(x - 6a)(x - b) = 0$. When distributed, the equation is $x^2 - bx - 6ax + 6ab = 0$. When $x$ is factored from the second and third term, the equation is $x^2 - (6a + b)x + 6ab = 0$, which is not equivalent to $x^2 + (2b - 3a)x - 6ab = 0$. 
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Question 39

Question and Scoring Guidelines
Question 39

Which property of dilations is sufficient to establish the AA similarity criterion for triangles?

A. preservation of orientation  
B. preservation of side length  
C. preservation of angle measures  
D. preservation of side length proportionality

**Points Possible:** 1

**Content Cluster:** Understand similarity in terms of similarity transformations.

**Content Standard:** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (G.SRT.3)

**Depth of Knowledge:** Level 1  
a. Recall, observe, or recognize a fact, definition, term, or property
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may recognize that dilations preserve orientation but did not consider that two triangles can have the same orientation without having the same angle measures.

Rationale for Option B: This is incorrect. The student may have thought that a dilation is a rigid motion and therefore preserve the side lengths.

Rationale for Option C: Key – The student correctly recognizes that dilations preserve angle measures and therefore are sufficient to establish the AA criterion for triangles.

Rationale for Option D: This is incorrect. The student may identify that dilations maintain the side length ratios and that this can be used to show similarity but did not realize that side length proportionality is not sufficient to establish the AA criterion alone.

Sample Response: 1 point

Which property of dilations is sufficient to establish the AA similarity criterion for triangles?

A. preservation of orientation
B. preservation of side length
C. preservation of angle measures
D. preservation of side length proportionality
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Question 42

Question and Scoring Guidelines
Question 42

A function \( f(x) = x^2 + bx - 20 \) has zeros at \( x = r \) and \( x = z \), where \( r \) and \( z \) are integers.

What is one possible value of \( b \) and what are possible values for the zeros of the function, \( r \) and \( z \)?

\[ b = \quad \]
\[ r = \quad \]
\[ z = \quad \]
Points Possible: 1

Content Cluster: Write expressions in equivalent forms to solve problems.

Content Standard: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (A.SSE.3)

a. Factor a quadratic expression to reveal the zeros of the function it defines.

Depth of Knowledge: Level 3

b. Explain thinking when more than one response is possible
f. Perform procedure with multiple steps and multiple decision points

Scoring Guidelines

Exemplar Response

- $b = 1$
  - $r = -5$
  - $z = 4$

Other Correct Responses

- any values for $b$, $r$, and $z$ where $r \cdot z = -20$ and $r + z = -b$

For this item, a full-credit response includes

- three correct values (1 point).
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Question 42

Sample Responses
Sample Response: 1 point

A function \( f(x) = x^2 + bx - 20 \) has zeros at \( x = r \) and \( x = z \), where \( r \) and \( z \) are integers.

What is one possible value of \( b \) and what are possible values for the zeros of the function, \( r \) and \( z \)?

\[
\begin{align*}
    b &= 1 \\
    r &= -5 \\
    z &= 4
\end{align*}
\]
Notes on Scoring

This response earns full credit (1 point) because it shows a correct possible value of one coefficient and the correct possible values for the zeros of the given function.

There is more than one way to approach this question. One way is to rewrite the equation of the function in factored form, which highlights its zeros. A quadratic function $f(x) = ax^2 + bx + c$ can be written in the factored form as $f(x) = (x - r)(x - z)$, where $(-r) \cdot (-z) = c$ and $(-r) + (-z) = b$. In this situation, $(-r) \cdot (-z) = -20$, or $r \cdot z = -20$ and $(-r) + (-z) = b$ or $r + z = -b$. The combinations for $r$ and $z$ can be any of the following pairs of integers: –5 and 4; 5 and –4; 10 and –2; –10 and 2; 20 and –1; or –20 and 1. For example, if the selected combination is $r = -5$ and $z = 4$, then $r + z = -1$, $-1 = -b$ and $b = 1$.

The function $f(x) = x^2 + 1x - 20$ written in factored form is $f(x) = (x + 5)(x - 4)$, where $b = 1$ and –5 and 4 are zeros.
Sample Response: 1 point

A function \( f(x) = x^2 + bx - 20 \) has zeros at \( x = r \) and \( x = z \), where \( r \) and \( z \) are integers.

What is one possible value of \( b \) and what are possible values for the zeros of the function, \( r \) and \( z \)?

\[
\begin{align*}
  b &= 8 \\
  r &= -10 \\
  z &= 2
\end{align*}
\]
Notes on Scoring

This response earns full credit (1 point) because it shows a correct possible value of one coefficient and the correct possible values for the zeros of the given function.

A quadratic function \( f(x) = ax^2 + bx + c \) can be written in the factored form as \( f(x) = (x - r)(x - z) \), where \((-r) \cdot (-z) = c\) and \((-r) + (-z) = b\). In this situation, \((-r) \cdot (-z) = -20\), or \(r \cdot z = -20\) and \((-r) + (-z) = b\) or \(r + z = -b\). The combinations for \(r\) and \(z\) can be any of the following pairs of integers: \(-5\) and \(4\); \(5\) and \(-4\); \(10\) and \(-2\); \(-10\) and \(2\); \(20\) and \(-1\); or \(-20\) and \(1\). For example, if a selected combination is \(r = -10\) and \(z = 2\), then \(r + z = -8\), \(-8 = -b\) and \(b = 8\).

The function \( f(x) = x^2 + 8x - 20 \) written in factored form is \( f(x) = (x + 10)(x - 2) \), where \(b = 8\) and \(-10\) and \(2\) are zeros.
Sample Response: 0 points

A function $f(x) = x^2 + bx - 20$ has zeros at $x = r$ and $x = z$, where $r$ and $z$ are integers.

What is one possible value of $b$ and what are possible values for the zeros of the function, $r$ and $z$?

$b = 1$

$r = 5$

$z = -4$

Notes on Scoring

This response earns no credit (0 points) because it does not show a correct possible value of one coefficient and the zeros of the given quadratic function.

The function $f(x) = x^2 + x - 20$ in the factored form is $f(x) = (x + 5)(x - 4)$, where $b = 1$ and $-5$ and $4$ are zeros, instead of $5$ and $-4$. 
A function \( f(x) = x^2 + bx - 20 \) has zeros at \( x = r \) and \( x = z \), where \( r \) and \( z \) are integers.

What is one possible value of \( b \) and what are possible values for the zeros of the function, \( r \) and \( z \)?

\[
\begin{align*}
  b &= 9 \\
  r &= -5 \\
  z &= -4
\end{align*}
\]
**Notes on Scoring**

This response earns no credit (0 points) because it does not show a correct possible value of one coefficient and the zeros of the given quadratic function.

The combinations for \( r \) and \( z \) can be any of the following pairs of integers: –5 and 4; 5 and –4; 10 and –2; –10 and 2; 20 and –1; or –20 and 1. The sum of two numbers within each pair is never equal to 9. For example, –5 + 4 = –1; 5 + (–4) = 1; 10 + (–2) = 8; –10 + 2 = –8; 20 + (–1) = 19; or –20 + 1 = –19, which means that over the integers, the expression \( x^2 + 9x – 20 \) is prime and –5 and –4 are not zeros of the function \( f(x) \).

The student may think that for a quadratic function \( f(x) = ax^2 + bx + c \) with zeros \( r \) and \( z \), the value of \( r \cdot z = –c \), but instead, \( (–r) \cdot (–z) = c \), or \( r \cdot z = c \). Since \( (–5) \cdot (–4) = 20 \) instead of \( –20 \) and \( (–5) + (–4) = –b \), so \( b = 9 \), the zeros for this function cannot be \( r = –5 \) and \( z = –4 \).
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Question 44

Question and Scoring Guidelines
Question 44

A scientist studies the population of an insect colony. She begins her study with 10,000 insects in the colony and measures the population of the colony each year. She uses the given formula to model the insect population, \( P \), \( t \) years since the beginning of the study.

\[ P = 10,000(1.08)^t \]

By what percent does the insect population grow each year?
Scoring Guidelines

Exemplar Response

- 8%

Other Correct Responses

- any equivalent percent

For this item, a full-credit response includes

- the correct percent (1 point).
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Question 44

Sample Responses
A scientist studies the population of an insect colony. She begins her study with 10,000 insects in the colony and measures the population of the colony each year. She uses the given formula to model the insect population, $P$, $t$ years since the beginning of the study.

$$P = 10,000(1.08)^t$$

By what percent does the insect population grow each year?

8%
The given formula models the growing insect population because the growth factor, 1.08, is greater than 1. The general formula for the exponential growth is \( P = N(1 + r)^t \), where \( N \) is the initial amount of the sample, \( r \) is percent growth or decay rate, expressed as a decimal, \( t \) is the time in years, and \( P \) is an amount of the sample measured \( t \) years after the initial amount \( N \) was measured.

The formula \( P = 10,000(1.08)^t \), can be restated to the equivalent form \( P = 10,000(1 + 0.08)^t \), to reveal the growth percent rate of the insect population, expressed as a decimal as 0.08. This is equivalent to the growth percent rate of 8% per year.
Sample Response: 1 point

A scientist studies the population of an insect colony. She begins her study with 10,000 insects in the colony and measures the population of the colony each year. She uses the given formula to model the insect population, $P$, $t$ years since the beginning of the study.

$$P = 10,000(1.08)^t$$

By what percent does the insect population grow each year?

$$\frac{16}{2} \%$$

Notes on Scoring

This response earns full credit (1 point) because it shows the correct yearly growth percent rate.

The student provides a yearly percent growth rate, $\frac{16}{2} \%$, which is equivalent to an 8% yearly growth rate.
Sample Response: 0 points

A scientist studies the population of an insect colony. She begins her study with 10,000 insects in the colony and measures the population of the colony each year. She uses the given formula to model the insect population, \( P, t \) years since the beginning of the study.

\[
P = 10,000(1.08)^t
\]

By what percent does the insect population grow each year?

\[
0.08 \text{ } 0\%
\]

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect yearly growth percent rate.

The student may forget to convert the decimal number 0.08 to the percent notation, or 8%.
Sample Response: 0 points

A scientist studies the population of an insect colony. She begins her study with 10,000 insects in the colony and measures the population of the colony each year. She uses the given formula to model the insect population, \( P \), \( t \) years since the beginning of the study.

\[
P = 10,000 (1.08)^t
\]

By what percent does the insect population grow each year?

1.08

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect yearly growth percent rate.

The student may incorrectly think that in the formula \( P = 10,000 (1.08)^t \), the entire base, 1.08, of the exponential expression \((1.08)^t\) represents a yearly growth percent rate.
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Question 45

Question and Scoring Guidelines
Question 45

A university determined the number of students pursuing different degrees, by gender. Some of the results are shown.

<table>
<thead>
<tr>
<th>Undergraduate Degree</th>
<th>Master’s Degree</th>
<th>Doctoral Degree</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td>500</td>
<td>12,500</td>
</tr>
<tr>
<td>Female</td>
<td>2,500</td>
<td></td>
<td>12,500</td>
</tr>
<tr>
<td>Total</td>
<td>16,250</td>
<td>6,250</td>
<td>25,000</td>
</tr>
</tbody>
</table>

What is the probability that a female student chosen at random is pursuing an undergraduate degree?

A  18%
B  32%
C  36%
D  64%

Points Possible: 1

Content Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Content Standard: Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. (S.CP.6)

Depth of Knowledge: Level 2
i. Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps
j. Translate between tables, graphs, words and symbolic notation
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly add the number of female students pursuing a master’s degree, 2500, to the number of female students pursuing a doctoral degree, 2000, \((2500 – 500 = 2000)\) to get 4500, divide it by the total number of students, 25000, and then identify the probability to be \(4500/25000 = 0.18\) or 18%.

Rationale for Option B: This is incorrect. The student may find the number of female undergraduate students to be 8000, \((12500 – 2500 – 2000 = 8000)\), divide that by the total number of students (25000) and then incorrectly identify the probability to be \(8000/25000 = 0.32\) or 32%.

Rationale for Option C: This is incorrect. The student may incorrectly add the number of female students pursuing a master’s degree, 2500, to the number of female students pursuing a doctoral degree, 2000, \((2500 – 500 = 2000)\) to get 4500, divide it by the total number of female students, 12500, and then identify the probability to be \(4500/12500 = 0.36\) or 36%.

Rationale for Option D: Key – The student finds the number of female doctoral students (second row/third column) to be 2000 \((2500 – 500 = 2000)\); the number of female undergraduate students (second row/first column) to be 8000 \((12500 – 2500 – 2000 = 8000)\), and then determines the probability that a female student chosen at random is pursuing an undergraduate degree as \(8000/12500 = 0.64\) or 64%.
Sample Response: 1 point

A university determined the number of students pursuing different degrees, by gender. Some of the results are shown.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Undergraduate Degree</th>
<th>Master’s Degree</th>
<th>Doctoral Degree</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td>500</td>
<td>12,500</td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td>2,500</td>
<td></td>
<td>12,500</td>
</tr>
<tr>
<td>Total</td>
<td>16,250</td>
<td>6,250</td>
<td>2,500</td>
<td>25,000</td>
</tr>
</tbody>
</table>

What is the probability that a female student chosen at random is pursuing an undergraduate degree?

A  18%
B  32%
C  36%
D  64%
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Question 47

Question and Scoring Guidelines
Question 47

Right triangle FGH is shown, where $\angle G$ is not equal to $\angle H$.

Which statement correctly describes the relationship between angles in triangle FGH?

(A) $\sin(\angle G) = \sin(\angle H)$  
(B) $\sin(\angle G) = \cos(\angle H)$  
(C) $\cos(\angle G) = \cos(\angle H)$  
(D) $\sin(\angle G) = \cos(\angle F)$

Points Possible: 1

Content Cluster: Define trigonometric ratios and solve problems involving right triangles.

Content Standard: Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)

Depth of Knowledge: Level 1
a. Recall, observe, or recognize a fact, definition, term, or property  
n. Represent math relationships in words, pictures, or symbols
Scoring Guidelines

Rationale for Option A: This is incorrect. The student may notice that angles \( G \) and \( H \) are complementary but incorrectly think that complementary angles have the same sine.

Rationale for Option B: Key – The student correctly recognizes that angles \( G \) and \( H \) are complementary and that the sine of \( G \), or \( \frac{FH}{GH} \), is equal to the cosine of \( H \), or \( \frac{FH}{GH} \).

Rationale for Option C: This is incorrect. The student may notice that angles \( G \) and \( H \) are complementary but incorrectly think that complementary angles have the same cosine.

Rationale for Option D: This is incorrect. The student may remember that in the right triangle the sine and the cosine of two angles are equal but miss that the two angles need to be complementary.
Sample Response: 1 point

Right triangle FGH is shown, where \( \angle G \) is not equal to \( \angle H \).

Which statement correctly describes the relationship between angles in triangle FGH?

\[ \text{A} \quad \sin(\angle G) = \sin(\angle H) \]

\[ \text{B} \quad \sin(\angle G) = \cos(\angle H) \]

\[ \text{C} \quad \cos(\angle G) = \cos(\angle H) \]

\[ \text{D} \quad \sin(\angle G) = \cos(\angle F) \]